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# A Systems Analysis of Grain Reserves

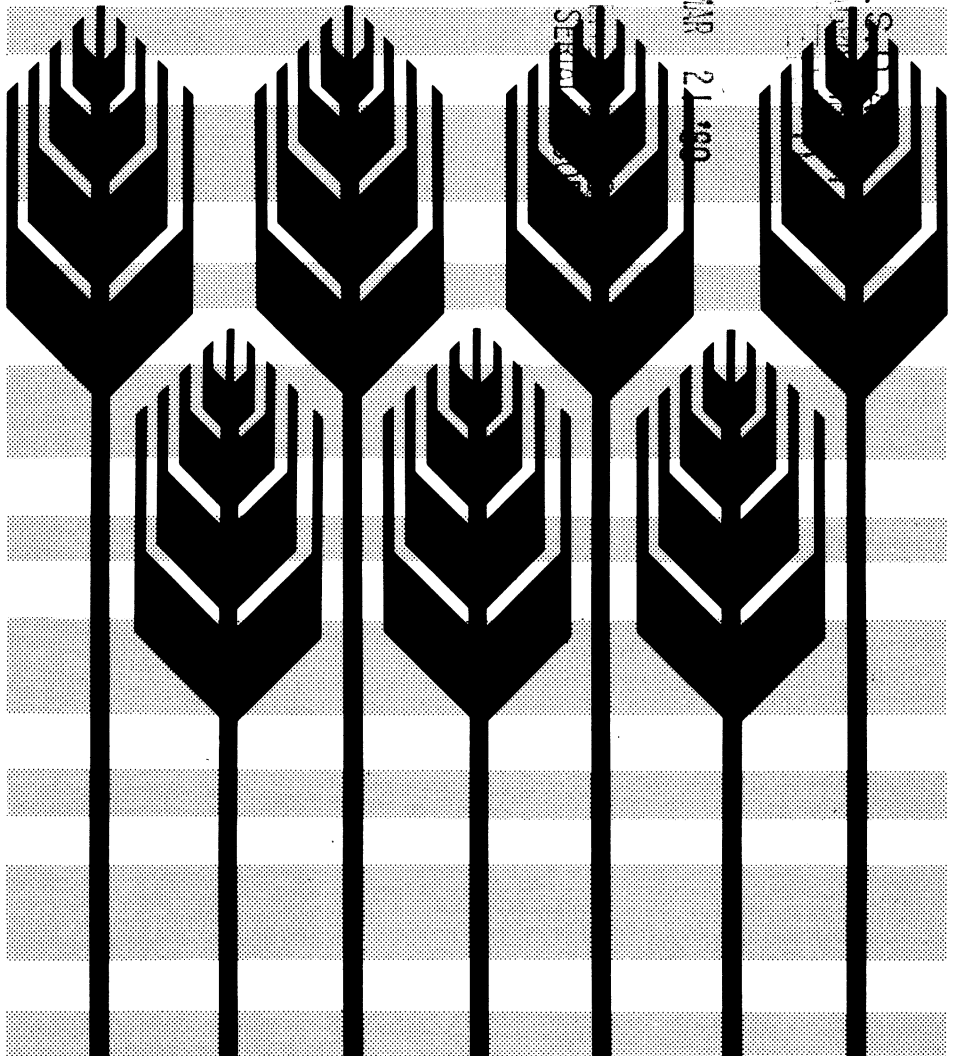
David J. Eaton

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A SYSTEMS ANALYSIS OF GRAIN RESERVES. By David J. Eaton.  
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#### ABSTRACT

The author develops techniques (1) to size and operate grain reserves to achieve multiple public objectives, (2) to assess buffer stock requirements to stabilize supplies, and (3) to evaluate buffer stock reliability. World grain supplies could fall as much as 5.5 percent below trend in 1975-2000 if there is no grain reserve and if production patterns mimic recent history. There is a tradeoff between the size of a reserve and its ability to provide grain in shortfall years — the more stability desired, the more grain needed in the buffer stock. There is no "optimal" grain reserve; the desired buffer stock will reflect judgments based on the importance of competing public objectives.

Keywords: Grain, grain reserves, world food security, systems analysis, demand, time series, mathematical programming, agricultural policy.

## FOREWORD

The uncertainty of crop production has meant that control of a food reserve could change the stability of prices or even governments. Management of buffer stocks by institutions in a democracy must account for the interests of many groups. This report develops methods to incorporate production fluctuations and public objectives into grain reserve management techniques. The report is part of a continuing effort of the International Economics Division to assess the impact of weather instability and production fluctuations on world grain supply and demand.

David Eaton conducted the research under contract with the Economics, Statistics, and Cooperatives Service while at The Johns Hopkins University (he has since moved to the LBJ School of Public Affairs, The University of Texas at Austin). Jared L. Cohon and Charles S. ReVelle, The Johns Hopkins University, and W. Scott Steele, Office of the Secretary of Agriculture, provided general supervision of this study. John Murray, formerly with ESCS, provided reviews.

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## SUMMARY

The author develops techniques (1) to size and operate grain reserves to achieve multiple public objectives, (2) to assess buffer stock requirements that would stabilize supplies, and (3) to evaluate buffer stock reliability. The techniques are used to find a lower bound on the size of a world buffer stock that would stabilize grain supplies from 1975 to 2000.

Grain supplies in a lean year during 1975-2000 could fall as much as 5.5 percent below the historical trend if future production patterns mimic recent behavior and if no grain reserve exists. There is a tradeoff between the size of a reserve and its ability to provide grain during sequential lean years — the more stability desired, the more grain needed in the buffer stock. There is no "optimal" grain reserve size. Rather, the size of global reserves will reflect judgments based on the importance of competing public objectives.

Following a background discussion that focuses upon the intended contributions of the research, the author presents and evaluates several historical analyses of grain reserves. He identifies issues underlying the grain reserve problem, including grain demand, the grain production process, and the international trade context of a buffer stock. After presenting mathematical methods an analyst can use in designing buffer stocks, the author generates synthetic grain production futures; formulates reserve sizing models that can incorporate objectives of supply stability, price stability, farmer goals, consumer interests, and net economic benefits; assesses reliability of a buffer stock; and develops ways to ascertain operating rules.

# A Systems Analysis of Grain Reserves

David J. Eaton \*

## I. — INTRODUCTION

One of the first acts of any permanent human settlement is likely to be the creation of a grain reserve. Early humans, as hunters and gatherers, moved with the seasons and their prey. Agriculture, and the implicit security of a renewable source of food, led people to settle in one place. The discovery of the buffer stock idea encouraged permanent human settlement.

Throughout recorded human history, a community's grain reserve has served as a source of social, economic, and political power. The role of a reserve in mediating supply and price fluctuation will remain as long as weather can influence the security of food production. Because grain stocks represent a political issue, researchers should explore the impacts of grain reserve policies. Specifically, the objectives are to:

- Examine multiple objectives which reflect economic and political goals,
- Design a reserve to stabilize supplies through a series of back-to-back lean years as well as isolated shortfall years,
- Characterize how grain production fluctuates over time,
- Include supply and demand interactions through a market, and
- Develop nonparametric procedures to assess result reliability.

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## EARLY GRAIN RESERVES

Both Joseph (the Biblical figure) and Li K'o (who lived in China in the 12th Century B.C.) needed to stabilize supplies of grain through good and lean years. Each devised a management strategy and each observed how grain supply and price fluctuations with a "no reserve" policy could cause social disruption.

### Joseph and the Pharaoh's Dreams

The most famous grain reserve is that of Joseph, son of Jacob, who lived in Egypt (1) 1/:

And Pharaoh spoke unto Joseph: 'In my dream, behold, I stood upon the brink of the river. And behold, there came up out of the river seven kine, fat-fleshed and well-favored, and they fed in the reed-grass. And, behold, seven other kine came up after them, poor and very ill-favored and lean fleshed, such as I never saw in the land of Egypt for badness. And the lean and ill-favored kine did eat up the first seven fat kine. . . .

Behold, seven ears came upon one stalk, full and good. And, behold, seven ears, withered, thin, and blasted with the east wind, sprung up after them. And the thin ears swallowed up the seven good ears.

Joseph interpreted the Pharaoh's dream to mean that 7 years of famine would follow 7 years of good crops. He instructed the Pharaoh to choose a man to set up a grain reserve that would contain one-fifth of each year's food production. The stored food would be used during the 7 years of famine.

The focus was not upon a single failure of production, but rather the time series behavior of gluts and shortages. Although the grain reserve was to stabilize supplies over 14 years, the multiobjective implications were clear to Joseph (1):

And there was no bread in all the land; for the famine was very sore, so that the land of Egypt and the land of Canaan languished by reason of the famine. And Joseph gathered up all the money that was found

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1/ Underscored numbers in parentheses refer to references at the end of this chapter.



in the land of Egypt, and in the land of Canaan, for the corn which they bought; and Joseph brought the money into Pharaoh's house.

And when the money was all spent in the land of Egypt, and in the land of Canaan, all the Egyptians came unto Joseph, and said: 'Give us bread; for why should we die in thy presence? for our money faileth.' And Joseph said: 'Give your cattle, and I will give you bread for your cattle, if money fail.' And they brought their cattle unto Joseph. And Joseph gave them bread in exchange for the horses, and for the flocks, and for the herds, and for the asses; and he fed them with bread in exchange for all their cattle for that year.

The second year, the people sold themselves and their land to Pharaoh, in exchange for bread from Joseph. Then he said to the people of Egypt (1):

Behold, I have bought you this day and your land for Pharaoh. Lo, here is seed for you, and ye shall sow the land. And it shall come to pass at the ingatherings, that ye shall give a fifth unto Pharaoh, and four parts shall be your own . . . .

Joseph's use of food as a source of power showed that a grain reserve creates social consequences which cannot be measured by any market mechanism. Although the buffer stock stabilized supplies, it also earned revenue for the reserve authority, the Pharaoh. The stock served both humanitarian concerns and the quest of a regime to centralize economic power.

The effectiveness of Joseph's reserve strategy was demonstrated not in the first lean year but rather in the last year of a series of sequential lean years, when all was gone except hunger. The stock was sufficient to stabilize supplies over the length of the worst possible failure of production that could be anticipated on the basis of existing information, the Pharaoh's dreams.

### Li K'o and the Principles of Confucius

Huan-Chang Chen, detailing Li K'o's story, states that according to Confucian theory (3):

. . . the government should level prices by the adjustment of demand and supply, in order to guarantee the cost of the producer and satisfy the wants of the consumer . . . .

Li K'o's policy, as Minister of Wei, was to keep the price of grain low enough so it would not hurt consumers and high enough not to impoverish farmers. He reasoned that (6):

If the consumers were hurt, the people would emigrate, and if the farmers were hurt, the state would be poor. . . . When his (Li K'o's) scheme was carried out in Wei, he not only made the people rich, but also made the state strong. . . .

Li K'o, like Joseph, recognized the multiple objectives implicit in decisions on grain stocks. The Chinese recognized four different goals: the farmer's, the consumer's, the economic interest of society at large, and price stabilization.

His rules were more complex than Joseph's storage of one-fifth of the harvest. He distinguished three likely levels of surplus output, each yielding different amounts of grain from the same amount of acreage. For each one of the three, he advised the government to buy a certain share and store it. This share would limit available supplies and stabilize prices. Li K'o specified similar release rules for years when production shortfalls occurred (3).

Joseph knew the pattern of future production in advance; Li K'o did not. This difference suggests why the Chinese system accepts a stochastic (probabilistic) description of good and lean years. Certain classes of excess production and shortfall are described; the reserve would stabilize even the worst lean year that could occur on the basis of existing information. Li K'o seems to imply that these events might occur in sequence and that the reserve system should provide grain even during such catastrophes. Li K'o, as did Joseph, notes that social dislocation is the real cost of the absence of a reserve system (3).

#### ISSUES IN GRAIN RESERVES

This report focuses on a global stock of grain, rather than a national or regional reserve or a collection of national stocks. All food and feed grains are considered,

including wheat, corn, rice, sorghum, oats, rye, and barley. Perfect substitution between all grains for all uses is assumed. Free trade is assumed to exist; there are no tariffs, export, or import controls.

Many important issues will not be considered, such as who pays for a reserve, who makes decisions, where the reserve should be located, and what form of international treaty arrangements are appropriate for policy implementation.

These assumptions are made to allow the development of a model of grain reserve sizing and operation that can be applied to assess policy issues. The assumptions can be dropped or changed, which would complicate the analysis further. They were chosen to illustrate the use of the model and to generate results that understate the size of a realistic world reserve. For example, if trade between nations was assumed to be prohibited, every country could maintain a reserve to stabilize domestic grain supplies. The aggregate stock would be much larger than a free trade global reserve. With a free trade global reserve, a bumper crop in one country can offset the shortfall in another; such inter-nation transfers could not exist if commerce is excluded.

Similarly, a reserve for all grains with perfect substitution assumed is smaller than separate stocks for each grain to stabilize supplies. If transportation networks did not exist and each country maintained its own stock, the aggregate of national stocks would exceed a single global reserve with such distribution.

In short, one way to view this model of grain reserve sizing is that it calculates a lower bound on reserve size. Relaxing the assumptions would be a more realistic reflection of existing practices and lead to a larger aggregate stock.

Four varieties of reserves can be distinguished. The working stock represents a marketing reserve in which grain is stored for gradual use over the remainder of the growing cycle. The buffer stock holds grain from a year of excellent harvests for use in another year plagued by poor production. Grain set aside at any time (even during a year of productive shortfall) for distribution to domestic or foreign persons defined as needy is a food aid reserve. An emergency food reserve is used in dispatch to a site of natural disaster or civil disorder.

Table 1 - Four types of grain reserves

Reserve type	Demand certainty	Rate of reserve turnover	Social function
Working stock	Relatively certain	Disposed of within 1 year of growing cycle	Intra-year stabilization
Buffer stock	Uncertain	Buildup and release rules set time in storage	Inter-year stabilization
Food aid reserve	Relatively certain	Determined by need of target group	Political leverage and/or humanitarian use
Emergency reserve	Uncertain	Depends upon what is defined as an emergency	Humanitarian use and/or political leverage

Conceptually, these four reserve varieties can be distinguished by the certainty of demand, the rate of stock turnover, and their function, as is done in table 1. As an institutional matter, the stocks are not so easy to distinguish. For example, the U.S. stocks held by the Commodity Credit Corporation have probably been used as each type of reserve.

The distinctions between the four types of reserves are drawn here to isolate the buffer stock from other types of reserves. While the other stock varieties are worthy of study, they are viewed here as topics for future research.

Physical or fiscal factors which limit users' access to a buffer stock will not be examined here. These factors include: grain losses from rodents or fungus; lack of political or economic control of the grain; absence of a grain transportation and distribution network; or users' physical conditions that limit absorption of the grain's nutrients.

Forecasting demand for grain involves many factors, which makes unanimity unlikely as to what constitutes the correct concept for "demand." <sup>2/</sup> One model developed in chapter five treats the volume of demand as deterministic, and equal to the expected volume of production. Over time, demands balance with supplies and the distribution of prices and demands can be generated by the distribution of production volumes. The buffer stock problem could be solved independent of demand because the stock would balance only fluctuations of supplies over time. A second model in chapter five relaxes this assumption; demands become a function of price.

<sup>2/</sup> See (5), (7-9) for several approaches.

## THE PRODUCTION VARIATION PROBLEM

One historic justification for a buffer stock has been to maximize food security — to stabilize supplies through good or lean years of fluctuating production. The test of security is that the stock should exceed the cumulative deficits of the worst series of back-to-back lean years that could occur. The problem is how to define this volume of "worst" deficit, and it is treated in chapter six.

Grain production involves technological, biological, physical, economic, political, and meteorological factors. To develop a descriptive model that would predict production based on causative variables is difficult.

The Li K'o example illustrates a frequently used approach that views the future as imitator of the past. All factors that influence production are assumed to be reflected implicitly in the historical measures of yields or the total volume of production; production here is treated statistically. Yet grain production forecasts based solely on history can be inaccurate. The historical record is too short for unambiguous statistical characterization. Although some U.S. data series can be traced back for a hundred years, no accurate records of world total grain production are available before 1950. No consensus thus exists as to the "best" curve to fit (and thus describe) the historical fluctuations. Further, the many forces that determine world grain output would undermine the validity of any statistical predictions based on historical behavior.

Despite these limitations, historical data are used here as the basis for analyzing future policies and developing a definition of the worst multiple-year deficit in grain supplies. The approach used here is to rely upon techniques not commonly used in agricultural analysis: synthetic grain production generation (for simulating possible future grain production fluctuations) and order statistics (for estimating the size of the worst possible grain shortfalls). These methods are drawn from the field of water resources analysis.

The synthetic generation results are adapted to the simulation of annual total world grain production by assuming that: the volume of production in any year is related to an expected volume (a trend); farmers' expectations are based on the previous year's production; and weather effects

are random. The equation that generates production volumes is designed to yield many production series, each indistinguishable in terms of its expected statistical behavior from another. Although annual world total grain production volumes may fluctuate widely, the expected behavior of all sequences mimics the historical record.

Simulated series of back-to-back production shortfalls are used in this study to assess the reliability of a grain reserve to stabilize supplies over time. A single-year shortfall is the difference between demand for grains and the level of production in that year. If demands equal the expected volume of production (as in the two-objective model of chapter five), the single-year shortfall may be defined as the difference between the actual production volume and the expected production volume (trend) in a year. The worst aggregate deficit over a series of years is the largest sum of yearly deficits that occur in sequence. For any period of  $n$  years there will be one worst aggregate deficit. If an analyst simulates many series of future production (each series is  $n$  years long), he can obtain a set of  $n$  worst aggregate deficits, one per series. These deficits can be ordered by their magnitude.

In chapter six, these ordered series of shortfalls are used to assess the likelihood that a buffer stock can stabilize supplies over a series of lean years. This approach accepts that there is no way to "know" the real distribution of production fluctuations. It uses the theory of order statistics and data from the historical record to find the largest likely aggregate shortfall of grain supplies and the likelihood of greater back-to-back deficits.

#### MULTIPLE GOALS IN BUFFER STOCK MANAGEMENT

Any decisions on the size of global buffer stocks will be made within a context of competing political interests. A farmer might wish a reserve to maximize his profits. An economist might insist upon maximizing net economic efficiency benefits. A mother of a malnourished child might evaluate a reserve by its ability to stabilize the quantities of available supplies even in years of lean production. The government of an importing nation might wish to minimize the price deviations of grain on the world market. While each person has biases about priorities among these and other objectives, an independent analyst

should present the full range of options and the implications of any particular ranking of objectives.

Few previous studies have tried to develop techniques to size a reserve to achieve multiple objectives. Previous optimization studies have focused on supply stabilization and net economic benefits. Although simulations may contain multiple measures of buffer stock performance, they do not address the issue of optimal reserve size.

Joseph and Li K'o viewed a buffer stock as a government intervention to stabilize a sector of the economy. A modern manager might want to stabilize supplies and prices and place some lower limit on price to protect farmers and some upper limit on price to protect consumers. Or a manager might wish to size a reserve to maximize either farmers' revenues or profits.

Others view a buffer stock as a public investment. Here, the manager would like to maximize net benefits to the economy from the stock. The manager would also want the reserve to be as small as possible (all other factors considered), because each increase in size adds to the costs of establishing, maintaining, and operating the reserve.

### Food Security and Supply Stability

Food security is not easy to define in a way that can be used in analysis. Consider this definition: the condition where the amount of grain available for consumption in any year is equal to the expected volume of production in that year. <sup>3/</sup> Partial security, or a shortfall in supplies available for consumption, can be expressed in terms of the percent of expected production not available in a year. For example, imagine that a stock can buffer supplies to the level of expected production in all but 1 year of 25. In that worst year, let the stock be depleted, with total supplies (production plus storage) equal to 99 percent of the expected volume of production. Here, the stock could be said to provide 99-percent food security; in the worst year, the supply shortfall is 1 percent below expectation. By definition, a reserve will perform better (provide more

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<sup>3/</sup> In the long term, the expected volume of production must be equal to the expected volume of consumption (including losses), because storage neither creates nor destroys grain. Thus, the word "consumption" could substitute for production in the definition.

security) in any year other than the one(s) that defines the food security level.

To be secure, a stock needs to stabilize supplies through the worst series of shortfalls as defined by simulated sequences of future production. Reserve reliability will be defined as the likelihood that a stock will stabilize supplies to a predefined level.

For example, imagine that an international organization does not want world grain supplies to fall more than 1 percent below trend production at anytime in a 25-year planning horizon. A reserve of Q tons is 95-percent reliable if a stock will buffer annual supplies to 99-percent security in 95 of 100 simulations of future production.

### Price Stabilization

Brandow has developed several arguments for price stabilization as a buffer stock goal (9):

. . . dependable supplies and stable prices in the U.S. will encourage the long-range development of commercial grain exports, which both the nation and the producers want . . . .

Another possible benefit of grain reserves is more stable output and prices in the livestock industries. Instability of livestock production induced by variable supplies and prices of feed grains surely causes inefficient use of fixed resources, income variability for livestock farmers and processing firms, and unstable employment of labor.

. . . Instability of food prices is itself a disutility to consumers. A related consequence of instability probably is an impetus to general inflation. . . .

Two other possible benefits of stable grain prices are the utility of reduced risk to grain producers and more efficient grain production resulting from less uncertainty.

Cochrane and Danin also support price stability (4, pp. 23-24):

. . . world grain price variability seems likely to be as great, or greater, in the next decade than it was in the last. The problem of price instability in



the grains, with the appropriate lags in animal product prices, is not going away by itself, nor will it be wished away. . . . In periods of sharply rising farm and food prices, policy actions . . . are likely to include:

1. the imposition of ceiling prices on food products,
2. the further expansion of food programs to assist the poor (e.g., the food stamp plan),
3. the use of export limitations of both formal and informal types, and
4. sporadic attacks on the monopolistic practices of big business and big labor in the food industries.

In periods of falling farm prices and stable to declining food prices, policy actions in the United States are likely to include:

1. efforts to maintain or raise commodity loan rates,
2. the imposition of production controls,
3. the expansion of foreign food aid programs, and
4. the making of supplemental income payments to medium and small sized commercial farmers.

The basic policy issue confronting consumers and producers of grain products specifically, and food products in general, in the United States may be formulated as follows. Are those interests content to leave the world grain price instability problem untouched and deal with its domestic symptoms in the future in essentially the same ways as they have been doing in the past 10 years? Or do they wish to initiate an international grain reserve program with the capacity to affect some reasonable stability in international grain prices and thus reduce the pressure to implement countervailing, or compensating, domestic programs?

#### Farmers' and Consumers' Goals

Farmers are used to some fluctuations in grain prices, but they do not want prices to drop too low and they want some reasonable level of revenue or profit. The U.S. Govern-

ment has for years placed a floor under the price level of some grains through the loan program of the U.S. Department of Agriculture. One operational definition of the farmers' ideal grain reserve would be the stock size that maximizes the minimum price they receive on grain. A second, more direct objective would be to maximize farmers' revenues and/or profits from grain production.

Consumers do not want prices to be too high. One surrogate for the consumers' objective is the minimization of the maximum price of grain.

### A Reserve as an Investment

The buffer stock, seen as a public investment, can be evaluated in terms of its costs and benefits to society. Gross benefits of a buffer stock can be defined as consumers' willingness to pay for grain. Reserve costs are those incremental commitments of real resources associated with the storage of grain. An analyst can determine whether such an investment is efficient by evaluating the level of the reserve's net economic benefits (that is, the difference between gross benefits and costs).

Grain storage volume per se can be an objective of the stock operator. With no factors other than size to consider, a buffer stock agency would probably want to keep the reserve as small as possible, because each increment of reserve capacity adds to the costs of the reserve.

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## II. — ANALYSES OF GRAIN RESERVES

The twin problems in designing a grain reserve are how large it should be (capacity) and how to time the acquisition and release of grain to stocks (operating rules). The following brief review presents work of previous analysts and inherent limitations in dealing with the two design issues.

### PREVIOUS RESEARCH

#### Statistical Analyses

Many Egyptian, Chinese, Greek, Roman, British, Dutch, and Indian analysts have made statistical studies of fluctuations in grain production (19). H.W. Working, who studied private wheat stocks kept for speculative purposes between 1884 and 1931, made one of the first modern analyses (12, p. 9; 15, p. 3; 51; 52).

Wells and Fox reviewed U.S. grain yield and consumption figures as a basis for suggesting stock targets (45, p. 4; 49; 26, p. 5). Waugh studied time series data for yields, acreage, and demands to estimate reserve size goals (48). The Secretariat of the U.N. Food and Agriculture Organization used three different methods to assess desirable stock levels (6), one of which was an analysis of trend deviations of area, yield, and consumption statistics (10, p. 6; 42, p. 18). Sayre and others used expert opinion to estimate stock target levels (35; 45, p. 3). R. Johnson suggested reserve size goals on the basis of experience (17).

Steele evaluated fluctuations in aggregate world production and import statistics to see what volume of grain would insure against a single-year contingency resulting from a production shortfall (8; 37). Trezise reviewed production (exporting countries) and consumption (importing nations) data to postulate reserve requirements for exporting nations to meet single-year commercial contingencies and food aid needs (42).

#### Welfare Theory

Oi derived the effects on producer surplus of price stabilization when demands can fluctuate (24; 25). Waugh showed how stabilizing prices (when supplies can fluctuate)

affects the level of consumer surplus (46; 47). Massell's model of stabilization effects when either supply or demand instabilities can exist (22; 23) was refined by Turnovsky (43). Samuelson showed that producer and consumer welfare changes due to price stabilization depend upon the shape of the demand and supply functions, and they may differ from those derived by Oi and Waugh (32; 33).

Subotnik and Houck compared effects of stabilizing price with the implications of stabilizing consumption or production (39). Changes in welfare were shown to depend on whether supply or demand is the source of instability; whether production or consumption is stabilized; and whether the market responds instantaneously or based on price expectations.

Sarris showed how welfare effects of price stabilization are calculated under assumptions of perfect or imperfect information (34).

#### Multimarket Simulation

Ray and associates developed POLYSIM, a computer simulation of supply, demand, and price formation activities in the feed grain, wheat, soybean, cotton, livestock, egg, and milk markets (27). Output variables include price, production, and consumption levels of each market sector, net farm income, government program costs, and consumer prices. Ray and others have used a version of POLYSIM to investigate changes in farm economies due to modifications in grain target price levels (9) and to simulate policy provisions (target price, loan rate, etc.) in Senate Bill 2005 (28; 9; 7, pp. 45-55; 26).

Levis and associates have constructed a 17-sector simulation of the U.S. agricultural economy, called AGRIMOD (20). The model simulates supply, demand, and price formation activities for many agricultural input factors and commodities organized through farm input, farm output (crop), and retail food trade markets. The model has been used to simulate the wheat trade during 1955 through 1970, and, through use of inferred government policies and price triggers, to generate or deplete stocks (20).

Takayama and Hashimoto have built a 1-period, 8-commodity, 20-region spatial equilibrium model of world food production, consumption, and trade (40; 41). They compare simulated behavior of world agricultural commodity markets during 1973-75 with historical data.

## Grain Reserve Simulations

Gislason derived two storage rules from theory and estimated coefficients from data for 1926-40 (11). These two rules, profit maximization or net economic efficiency maximization, were simulated for a corn market with a stationary linear demand function and production volumes for 1926-37. Resulting carryover levels were compared to actual stock volumes.

Baily, Kutish, and Rojko simulated world cereal markets to observe the effects of a stock (defined as a percentage of U.S. production) upon supply and price levels (1). Supply shortfalls, with and without U.S. stocks as buffers, were calculated for seven world regions based on historical production data for 1950-70.

Reutlinger developed a model of the world wheat market to show the implications of a world buffer stock upon net economic efficiency benefits, the profitability of a reserve authority, and welfare shifts between producers and consumers (29; 30). Demand was defined as a stationary, piecewise-linear function. Grain production was a stationary, independent stochastic process with a triangular distribution. Price rules dictate reserve behavior: if prices go above one level, they trigger releases; if prices fall below another level, they trigger acquisitions. World wheat market behavior was simulated over 300 30-year investment cycles so that the expected values and standard deviations of output variables could be evaluated.

Reutlinger, Eaton, and Bigman extended this model to simulate a system of interacting world and national grain markets for evaluating the desirability of a less developed nation's investment in grain reserves (31). World and national production were defined as stationary, independent stochastic processes with normal distributions, and the model could use other distributions or correlated production processes. World and national demands were described as piecewise-linear stationary functions. Stock decisions could be triggered either by volumetric or price rules. The model calculated the effects of a country buffer stock on domestic net economic efficiency, producer income, consumer expenditures, foreign trade balances, indices of shortfall frequency, and buffer stock profitability.

Walker and Sharples' model of the U.S. wheat market shows how a stock could affect distribution of wheat prices, supplies, reserve costs, and Government deficiency payments for 1975-81 (36). Supply was a function of the previous

year's price, a trend factor, and a random disturbance. Domestic and export demands were composed of price and trend factors; export demand was further affected by a random disturbance term. Alternate price rules for a buffer stock were tested with 300 simulations for 1975-81.

Keeler's variant of the Walker and Sharples model tests different stock rules and computes additional measures of benefits (18). He considered the effects of privately held grain stocks and Government subsidies for private stocks.

Brzozowski developed a systems dynamics model of the U.S. wheat market which calculates inventories, domestic consumption, production, and exports for alternate scenarios (3). Model results were compared with actual wheat market performance of 1972-74. The effects of a U.S. drought during 1975-76 were also simulated.

Tweeten and associates modeled the 1970 U.S. wheat market to observe how buffer stocks affect prices, farm receipts, net farm income, and net social costs (44). Production was a stationary, independent stochastic process of empirical distribution. Demand was built from foreign and domestic components, with export volume influenced by a random shock factor. Three storage rules were used: stock volumes as a function of price; upper and lower price bounds to trigger release or acquisition of grain; and a rule derived from dynamic programming to minimize a net social loss function.

Winter and Iga developed storage rules to balance the costs of carrying Canadian wheat with benefits of stocks for "pipeline" and commercial contingency purposes (50). The rules were simulated for historical (1944-69) Canadian wheat production, and carryover was compared to the historical levels. The rules were also applied to a set of randomly generated yield and demand patterns.

Cochrane and Danin developed a simulation for both the world and U.S. grain trades for 1975-85 to investigate how reserve stocks stabilize prices and supplies, and how these results compare to free market behavior (4). The demand function was composed of a trend factor, a random disturbance term, and price effects. The supply function was built from a trend effect, the cobweb impact of previous year's price, and a random shock. Two grain stock rules were developed to stabilize prices over time: a bounded price rule and a price variability minimization rule. The

former limits price fluctuations beyond a prescribed bound. The latter attempts to reduce an index of price variability.

### Dynamic Programming Models

A number of analysts have built dynamic programming models based on Gustafson's work which maximizes net economic efficiency benefits (13). Gustafson assumed demand to be strictly domestic and defined it as a function of price. Production was a stationary, independent stochastic process of known distribution. The storage rule was that function which maximizes net economic efficiency.

Johnson and colleagues have applied Gustafson's methods to find storage policies for less developed countries (15, pp. 17-30; 16). Stock rules and levels were determined under assumptions of free trade, restricted trade, and various demand elasticity assumptions.

Sarris reformulated Gustafson's objective function as a weighted average of producer and consumer benefits (34, pp. 14-16). He developed the explicit form of this multi-objective function for linear and nonlinear demand functions, but did not estimate or use them with data. Nor did he mention the computational difficulties of using dynamic programming to solve a two-objective function.

### EVALUATION OF PREVIOUS RESEARCH

We can now isolate the reasons for developing a new methodology for grain reserve analysis. To some degree, the goal of analysis determines its technique. In this report, the purpose is to design a grain reserve that will achieve explicit social objectives. The twin design problems are capacity and operating rules. 4/ The objectives are not limited to efficiency or stabilizing supplies, but can include any goals advocated by an interested party. Each available technique has limitations for dealing with these issues.

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4/ These two problems are closely related. They will be treated as separate issues for the purpose of methodology development.



## Statistical Analyses

There are major differences between the conception of the "grain reserve" problem as defined here and in many of the statistical analyses. Previous studies have usually sized a stock to stabilize supplies through an isolated fluctuation of production below trend — a single year's shortfall. This report extends stabilization to a series of year-after-year lean harvests. Statistical studies often review a series of historical production events. Here, the analysis is extended to a prospective future.

Some studies rely upon expert judgment to determine future production levels. Others assume a distribution of production fluctuations. Some do recognize that there is no way of knowing the "real" distribution of production volumes.

In some studies, results are evaluated through confidence intervals. Often, the word "probability" is associated with results, as in (42, p. 17):

. . . at the 95 percent confidence interval, the probability is one in twenty that variations from trend will be as large or larger than the calculated figure. In order to cover a specific risk up to 95 percent probability, the reserve target would have to be set at the corresponding level shown in the calculations.

Such reasoning is imprecise, as the calculated intervals are sample statistics and not population parameters; they thus only estimate some confidence interval. The lower and upper bounds on the underlying interval are not fixed, but are random variables with probability distributions. This report uses an alternate nonparametric technique for evaluating result reliability that is independent of such distributions.

Some statistical studies design a reserve for a low production fluctuation that is known, and not for a shortfall larger than the largest that might occur. This report presents a procedure for design that uses the worst possible aggregate shortfall of supplies that could occur based on some existing information.

## Welfare Economics Theory

Welfare studies consider the effects of stabilization upon welfare shares. They have yet to involve the design

of a stock system to achieve particular share configurations, stability, or other objectives. Existing welfare economics results hold only for a narrow range of assumptions. Producer, consumer, and net economic benefits usually depend upon (1) what is being stabilized; (2) the source of the instability; (3) the shape of demand and supply curves; (4) perfect competitive market conditions; and (5) the assumption of costless, complete stabilization.

### Simulations

Simulation describes rather than prescribes. Given assumptions, a simulation calculates how the system will respond; we gain insight into the world through induction. But one cannot know whether an untested solution exists which better achieves the stated goals. Optimization is the preferred mode of analysis for screening many alternatives (5, pp. 14-22). Simulation can, however, be used in conjunction with any of the optimization models proposed in later chapters. The multisystem models have to date focused more on validation of the subcomponents and overall structure than on detailed policy analysis (20; 40; 41). Simulations do, however, include market processes. Supply and demand curves are often based on empirical considerations and prices adjust so that equilibrium is achieved.

### Dynamic Programming

Economic efficiency has been the sole objective of dynamic programming analyses of grain reserves (13; 15; 16). Such an approach implies that (1) producer or consumer welfare should not motivate policy and (2) price or supply stabilization, *per se*, should not be considered as legitimate social goals (14; 2; 21).

Dynamic programming could be used with other objectives. Gustafson, the original user of dynamic programming for grain reserve analysis, noted that his model could be modified to maximize farm welfare shares or benefits to other interests (13). Dynamic programming is not used here, although future research may be directed at using multiple objectives in a dynamic program.

## A RESEARCH AGENDA

Based on this review of research, a design-oriented grain reserve methodology should contain five capabilities. It should:

- . Consider explicitly multiple objectives which reflect economic and political goals;
- . Stabilize supplies through a series of back-to-back lean years as well as isolated shortfalls;
- . Characterize explicitly how production fluctuates over time;
- . Develop nonparametric procedures for evaluating result reliability and avoid reliance on confidence intervals; and
- . Include supply and demand interactions through a market.

The first four capabilities have been well developed by practitioners in the field of water resource systems. A review of work in that literature will assist in the development of these tools for grain reserve analysis.

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### III. — POTENTIAL TRANSFERABILITY OF WATER RESOURCES THEORY TO GRAIN RESERVES ANALYSIS

In grain reserves analysis, sizing and operating rules relate to the natural phenomenon of grain production. In water resources theory, streamflow is the comparable natural event while policy issues involve water reservoir sizing and operation. In this chapter, we review water resources techniques for possible transfer to analysis of buffer stocks. Three issues to be explored, defined in chapter two, are: how to characterize a time series of natural fluctuations; how to size and operate a reserve; and how to assess the meaning and limits of results.

#### CHARACTERIZATION OF TIME SERIES

Three procedures used to characterize streamflow events over time are historical series, descriptive simulations, and statistical characterization. Rippl used a history of cumulative streamflows as a surrogate for what might occur in the future (13; 18). <sup>5/</sup> The implicit assumption was that the future will repeat the exact data series of the present.

A descriptive simulation is a model of natural processes (such as rainfall and watershed response) which aims to mimic observed streamflow events. Functional forms for relations among variables would be assumed to estimate coefficients from observed data. Such a simulation could characterize future flows on either a deterministic or stochastic basis.

Many techniques have been developed to characterize streamflow series statistics. These methods include synthetic hydrology, time series analysis, fractional Brownian motion, and matrices of joint transition probability distribution.

Thomas and Fiering developed synthetic hydrology, which describes streamflow events as a function of a random shock, the deviation of flows in the previous period from expected behavior, <sup>6/</sup> and statistics calculated from a time series of historical streamflows (5; 21). The goal of

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<sup>5/</sup> This approach resembles early statistical analyses of grain reserves, as in (25).

<sup>6/</sup> Autoregressive behavior of lag one.

synthetic generation is to create a data series that is similar to an original historical record in some respects, but quite different from it in others. The actual values and their order of occurrence can be quite different. Assuming the process is stationary and homoscedastic, statistical measures which characterize the fluctuations of streamflow events about an expected value should be identical. Synthetic series thus become, for analytical purposes, a potential extension of the historical sampling record.

Each series has a mean, standard deviation, and lag-one autocorrelation coefficient equal to the original series. The mean and standard deviation should not change because streamflow patterns are assumed to have a constant mean and variance through time. The lag-one autocorrelation coefficient represents the relation of flows in the previous period to current levels.

Thomas and Fiering found that one function did generate series which maintained the mean, standard deviation, and lag-one autoregressive behavior of a process:

$$I_{t+1} = \mu + \rho(I_t - \mu) + \delta_{t+1}(1-\rho^2)^{0.5}\sigma \quad (3-1)$$

- where  $I_{t+1}$  = streamflow in period  $t+1$
- $\mu$  = the mean level of streamflow for all periods
- $\rho$  = the lag-one autocorrelation coefficient between flows
- $\delta_{t+1}$  = a random normal deviate for period  $t+1$   
 $\delta \sim N(0,1)$
- $\sigma$  = the standard deviation of streamflows for all periods

Time series analysis also uses empirical statistics from a time series of streamflows to forecast future flows, but it is more flexible and designed for different goals than synthetic hydrology (2). Time series analysis can describe flows with moving averages and autoregressive effects. It can thus be applied to describing nonstationary

or heteroscedastic patterns. 7/ Time series analysis is generally used to forecast a deterministic series rather than to simulate (2; 12).

Mandelbrot and Wallis proposed fractional Brownian motion for describing streamflow sequences (10). Such a model allows past events to affect current observations through a memory term, whose magnitude decays with time.

Loucks used matrices of joint transitional probability distribution to describe the pattern of streamflow (9). 8/ Historical flows were divided into classes on the basis of volume. He calculated the relative frequency for an event in a given class in the current period, given that flow in some class in the previous period was observed. The matrix of transitional relative frequencies was used in place of simulated flow histories.

In this study, the simulation of grain production futures is restricted by the available data base. In chapter eight, a data series of only 15 years of annual production is used (22) although a series of 26 has subsequently become available (23). The stochastic method least influenced by series length is synthetic hydrology. Because the data are so sparse, the use of the more sophisticated methods just described is not justified here (19).

## RESERVOIR DESIGN

Researchers in water resources have developed procedures to determine both the size and the operations rules for water reservoirs.

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7/ A stationary series is one whose expected value does not change over time. A series with a trend is not stationary. A heteroscedastic series is one whose variance changes over time. A homoscedastic series is one whose variance remains constant over time.

8/ Loucks did not intend his representation of streamflow events to be an alternative to synthetic generation. The transition probability approach followed from his goal of including the stochastic behavior of streamflow explicitly in an optimization model.

## Sizing Techniques

Rippl's technique sizes a water reservoir based on an historical series of water flows (13; 18). The inputs are a plot of cumulative runoff as a function of time and a time rate (constant) of dam draft (water demands). They determine the smallest reservoir size which can meet the level of draft given a streamflow regimen.

The method of sequent peak, another computational technique, uses an empirical time series (13; 26). Given the history of flows and a deterministic series of drafts, the analyst calculates the difference between draft and inflow, called tentative storage. Patterns of storage determine the smallest reservoir volume which can provide water at the defined release levels.

A linear programming formulation achieves the same result as the Rippl or sequent peak methods (14; 26). The data inputs are a deterministic series of water demands and a real or synthetic history of streamflow events. The linear program determines the smallest reservoir size which provides for the water demands by storing from the fluctuating flows while maintaining continuity (Table 2).

Roefs described a dynamic programming formulation which is roughly comparable to the previous linear program (7; 11; 19). A pattern of water inflows is required as data. The options for possible levels of reservoir storage or releases are limited to discrete distributions. The formulation has a continuity constraint and boundary conditions for the reservoir volume or the level of benefits (from storage/release) at the conclusion of the design period. Release and storage volumes are expressed as decision variables with discrete distributions. One objective function would be:

$$F(S_{t-1}) = \underset{[all R, S]}{\text{MAXIMIZE}} [v(R_{t-1}) + f(S_t)] \quad (3-8)$$

where  $S_t$  = storage in period  $t$

$R_t$  = release in period  $t$

$f(S_t)$  = the value of being at  $S$  in period  $t$

$v(R_t)$  = the value of being at  $R$  in period  $t$

Table 2 - Linear program for sizing a water reservoir

Concept	Verbal statement	Mathematical statement
Objective function:	Minimize the reservoir size	Minimize C (3-2)
Subject to the constraints:		
Continuity	Matter is neither created nor destroyed	$S_t = S_{t-1} + I_t - U_t$ in all periods $t, t=1,2,\dots,n$ (3-3)
Capacity	Capacity is less than or equal to the reservoir size	$S_t \leq C$ in all periods $t, t=1,2,\dots,n$ (3-4)
Boundary conditions	Storage at beginning and end of the planning horizon is defined and equal	$S_0 = S_n$ (3-5)
Demand	Release exceeds or equals demand	$U_t \geq O_t$ in all periods $t, t=1,2,\dots,n$ (3-6)
Non-negativity	All variables are greater than or equal to zero	$S_t, I_t, U_t, C \geq 0$ in all periods $t, t=1,2,\dots,n$ (3-7)

where:

$S_t$  = storage in period  $t$  (unknown)

$I_t$  = water inflow in period  $t$  (unknown)

$U_t$  = water outflow in period  $t$  (unknown)

$O_t$  = demand in period  $t$  (known)

$C$  = capacity of the reservoir (unknown)

Both simple linear and dynamic formulations assume a deterministic inflow series, whether historical or synthetic. Loucks' linear programming method uses matrices of transition probabilities for determining both reservoir size and operating policy in a single step (9). The modified continuity constraint says that total releases over all time from the reservoir shall equal the water inflow. A function is defined which associates a value with any given release while the reservoir is at some volume, given some inflow for a time period. The program maximizes these benefits. Such a formulation has been termed "linear explicit stochastic programming" (8). Ahmed presented a model where the value of releases could not be expressed as the sum of linear terms, which he resolved with dynamic programming (1). His model is known as "dynamic explicit stochastic programming" (8).

ReVelle and others developed another approach to joint determination of optimal reserve size and operating rules (16). Information requirements included an explicit form for the reserve release rule (linear) and a cumulative distribution of historical streamflows. Chance-constrained linear programming is used to solve this problem. The method has been termed "the linear decision rule." Numerous extensions of this work exist (4; 6; 8; 15; 17).

The choice of a reserve sizing technique that can be applied here to buffer stocks is restricted by the data base and the desire for multiple objectives in the formulation. As mentioned, the limited historical record of grain production makes it difficult to justify the use of the most sophisticated techniques.

The choice between the simple linear or dynamic formulations rests upon computational considerations. Dynamic programming could be used if there was only one linear or nonlinear objective. However, there are likely to be many objectives for buffer stocks which may be expressed as linear equations. More than a few decision/state variables or objectives impose serious computational burdens upon a dynamic program (24). Multiobjective linear programming can accommodate up to five or six objectives without imposing intolerable computational burdens (see chapter 5). Thus multiobjective linear formulations related to equations (3-2) through (3-7) are most suitable for this grain reserve analysis.

## Operating Rules

The linear program formulated as equations (3-2) through (3-7) determines a minimum size for a reserve.<sup>9</sup> Associated with that analysis is a technique for selecting reserve operating rules (26). If capacity is minimized subject to constraints (3-3) through (3-7) and to input streamflows, the result will be an optimal capacity and a set of storage volumes, one for each period (the vector of  $S_t$ ). The decision either to store or release water from the reservoir can be read from the values of  $U_t$  and  $I_t$ :

$$S_t - S_{t-1} = I_t - U_t \quad (3-9)$$

Whenever  $I_t$  is less than  $U_t$ , a release decision has been specified. If  $I_t$  exceeds  $U_t$ , an increment to storage results from the excess of inflow above the release level. If the sequence of streamflows would be certain to repeat exactly, the vector of decisions ( $U_t, t=1,2,\dots,n$ ) would be optimal. They would be optimal because they would give rise to the smallest required reservoir capacity which can meet water demands ( $O_t, t=1,2,\dots,n$ ).

But what if the future is not identical to the past? Here, synthetic generation can be used. Any number of time series of streamflow patterns can be generated, each with expected value, variance, and lagged covariance behavior identical to the historical sequence. A set of synthetic streamflow sequences {sequences=1,2,...,b}, each of n years of length {t=1,2,...,n} can be considered a random sample of size b, drawn by random number generation from the probability distribution of all sequences of n years. <sup>9/</sup> Let each of the b series be used as flow input to the linear program of equations (3-2) through (3-7). When the linear problem is solved, its results would be:

- . b capacities, each of which is the smallest reserve size capable of providing water at the draft rates given the streamflow regimen, and

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<sup>9/</sup> More accurately, this is a pseudo-random sample of size b because a computer cannot generate random numbers — just numbers that appear to be random and are thus termed "pseudo-random." The nonrandomness in existing random number generators does not appear until many numbers are generated. For the sequence lengths discussed here, a pseudo-random sample could not be distinguished from a random sample.



- . b sets of n storage/release decisions (one for each year), each of which is optimal for associated streamflows.

Both results are of analytical interest. In chapter 6, the set of capacities will be used in a procedure to assure the reliability of results. Here, let us examine how the b sets of n storage/release decisions can be used for decision rules for reservoir releases.

For every year in every sequence, there is a streamflow volume, a level of storage in the reservoir, and a decision in that year to add to or release water from the reservoir. One form of a decision rule for a reservoir is to relate these variables. In other words, the decision to release can be based on:

- . The streamflow in the period or in some previous period,
- . The level of water in storage in the period or in some previous period, or
- . The storage/release decision in some previous period.

The problem of determining an operating rule has now been reduced to the technical issue of finding a relationship among release decisions, streamflow, and storage. Regression analysis can be used. Clearly, a large number of relationships among these variables is possible. An example would be where the release decision in some period is a linear function of the streamflow in that period, as calculated through a least squares regression.

## HOW TO ASSESS THE RELIABILITY OF RESULTS

Thomas applied the theory of order statistics to the problem of determining the proper capacity for a flood control dam (20). Imagine a record of the largest annual floods over n years of history. These n floods can be ordered from largest to smallest:

$$Y_{(1)}, Y_{(2)}, \dots, Y_{(m)}, \dots, Y_{(n)} \quad (3-10)$$

where  $Y_{(k)}$  is the  $k^{\text{th}}$  largest annual flood on record.

Imagine that an engineer wishes to design a flood control dam to contain a flood of the magnitude of the  $m^{\text{th}}$  (in order) flood. This volume is the design size of the flood control reservoir. The engineer may wish to know whether a flood will occur in the next year which will be larger than the design flood (the  $m^{\text{th}}$  largest in order of  $n$  floods).

Let  $g$  be defined as the true probability that the  $m^{\text{th}}$  in order of  $n$  floods will not be exceeded by a flood in the next year. The engineer cannot know the value of  $g$ , as it is a population parameter and he only has a small sample of flood behavior ( $n$  years). Let  $g_0$  be defined as an estimate of  $g$ , based upon the historical record. Let  $\Theta$  be defined as the probability that the actual  $g$ -value of the  $m^{\text{th}}$  flood will be less than the estimate,  $g_0$ . 10/

Thomas showed (20, p. 435) that the value of  $\Theta$  is:

$$\theta = \binom{n}{m} m \int_0^{g_0} g^{n-m} (1-g)^{m-1} dg \quad (3-11)$$

where  $\Theta$  = the probability that the actual  $g$ -value of the  $m^{\text{th}}$  flood will be less than the fixed value estimate,  $g$

$n$  = the number of floods in the record

$m$  = the order of the  $m^{\text{th}}$  flood

$g$  = the probability that the  $m^{\text{th}}$  in order of  $n$  floods will not be exceeded in the next year

$(1-g)$  = the probability that the  $m^{\text{th}}$  in order of  $n$  floods will be exceeded in the next year

Thomas also showed (20, p. 436) that the expected value of  $g$  is:

$$\bar{g} = \frac{n-m+1}{n+1} \quad (3-12)$$

where  $\bar{g}$  is the expected value of the parameter  $g$ .

10/ Note that  $\Theta$  is a probability measure of a probability measure,  $g$ . The true value of  $g$  is unknowable; there can only be sample estimates of  $g$ . Equations (3-11) and (3-12) do make it possible to make explicit statements in probabilistic terms about the behavior of  $g$ .

Equations (3-11) and (3-12) relate to the case where the engineer wants to know if the design volume will be exceeded in the next year by a large flood. Another issue is the likelihood that the  $m^{\text{th}}$  largest of  $n$  floods will be exceeded exactly  $k$  times in future years. Let  $\theta_{kt}$  be defined as the probability that in  $t$  future years the  $m^{\text{th}}$  of  $n$  past floods will be exceeded exactly  $k$  times. Again from Thomas (20, p 437):

$$\theta_{kt} = \binom{t}{k} \binom{n}{m} m \int_0^1 g^{t-k} (1-g)^k g^{n-m} (1-g)^{m-1} dg \quad (3-13)$$

which, when integrated, becomes:

$$\theta_{kt} = \frac{m \binom{t}{k} \binom{n}{m}}{(m+k) \binom{t+n}{m+k}} \quad (3-14)$$

Equations (3-11) through (3-14) will be applied in chapter 6 to develop measures of grain reserve performance reliability.

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#### IV. — SYNTHESIS OF GRAIN PRODUCTION SERIES

Synthetic generation of crop futures involves the selection of behavior to be reproduced and the development of a generating function. Behavior can be selected either to conform with assumptions or through the characterization of an historical data series (see chapter 3). Four categories of information are:

- . Expected volume of production,
- . Expected variance of production fluctuations about the expected volume,
- . Lagged covariance behavior of fluctuations, if any, and,
- . Form of the random behavior of fluctuations.

#### TIME SERIES BEHAVIOR

The synthesis approach accepts the notion that the four categories of information just listed are sufficient to describe how world grain production will behave over time. The expected volume is the average production volume. The variance measures the relative size of fluctuations about the expected volume. Lagged covariance behavior describes how fluctuations in 1 year appear to affect production in future years. Once the lag effects are isolated, the remaining fluctuations are attributed to random behavior, which can be characterized by a probability distribution. Once a behavior is selected, a function is found to mimic it. The function should create plausible values which may not have occurred, although the patterns of data behavior should be maintained.

#### Expected Volume

The concept of expected volume, when there is a trend, implies that production in any year  $t$  is related to the year  $t$ :

$$E[I_t] = f(t) \quad (4-1)$$

where  $E[I_t]$  = expected production in year  $t$

$f(t)$  = a function of year  $t$

One mathematical specification may be selected over others based on the fit with the historical record. Such fitting may be accomplished by computer routines for regression analysis. Table 3 lists forms an analyst may wish to test against the data.

Table 3 - Alternate trend hypotheses

Hypothesis of growth in production	Mathematical formulation
No trend	$E[I_t] = \mu$
Linear trend	$E[I_t] = \alpha + \beta t$
Exponential trend	$E[I_t] = \alpha \exp(\beta t)$
Quadratic	$E[I_t] = \alpha + \beta t + \gamma t^2$
Power	$E[I_t] = \alpha t^\beta$
Logarithmic	$E[I_t] = \alpha + \beta \ln t$
Sinusoidal	$E[I_t] = \alpha \sin(\phi + \beta t)$
Logistic	$E[I_t] = \delta \left[ 1 - \frac{1}{1 + \alpha \exp(\beta \delta t)} \right]$

where  $I_t$  = production in year  $t$

$\mu$  = a mean value

$\phi, \alpha, \beta, \gamma$  = coefficients

$\delta$  = an exogenous upper limit



## Variance

If a data series is short, it may be difficult to determine empirically how the variability of fluctuations changes with time. Several hypotheses may be considered:

- . Constant variance:      The magnitude of fluctuations does not change over time,
- . Constant coefficient of variation:      A special case of increasing or decreasing variance; the absolute magnitude of fluctuations may change over time, but is constant relative to changes in the expected volume, and
- . Increasing or decreasing variance:      The magnitude of fluctuations increases or decreases over time according to some rule.

Figure 1 illustrates constant, increasing, and decreasing variance behavior.

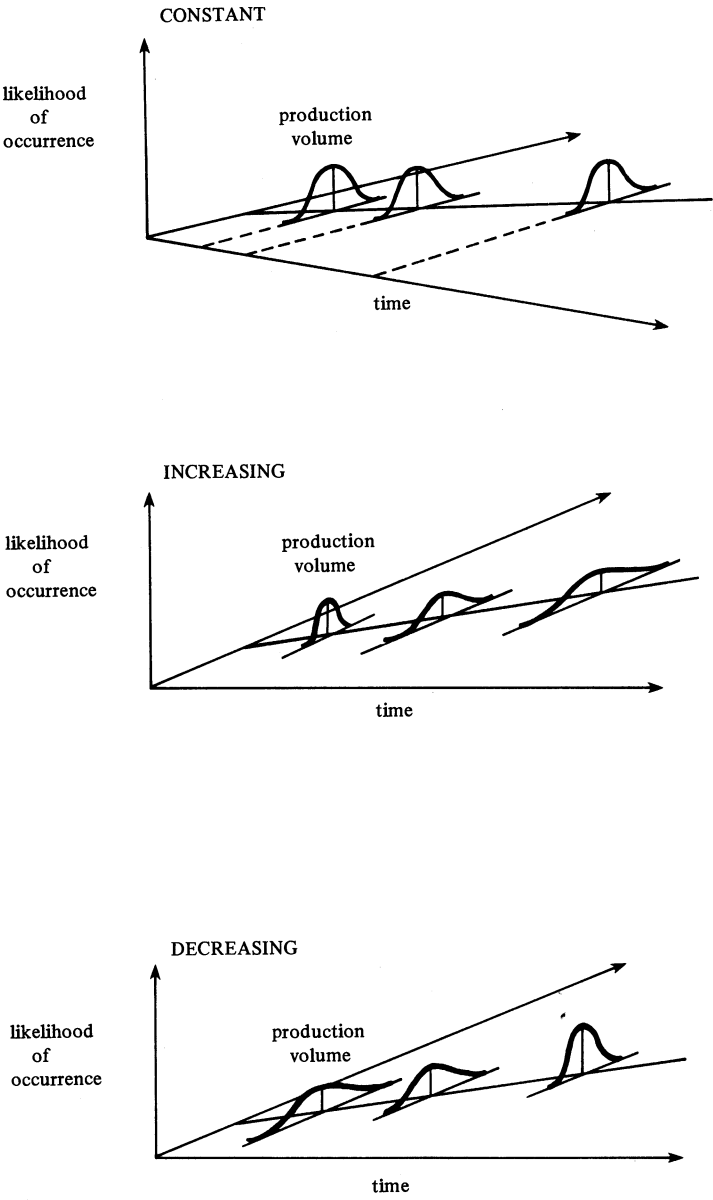
The selection of a variance assumption may reflect some empirical result or the analyst's view of his predictive capabilities. Variance can also correspond to perceptions of human control of farm production. Decreasing variability is consistent with a view of technological optimism: that man's control of natural forces is increasing and thus production fluctuations will decrease over time. Increasing variability could reflect ecological pessimism — that human reliance upon nitrogen fertilizers and pesticides may lead to periodic productive failures due to energy shortages, pest resistance, and increasing variability of natural weather patterns.

## Lagged Effects

The lagged covariance structure of production describes how a surplus or shortage in 1 year affects production in another year. There have been a number of hypotheses advanced to account for lagged fluctuations.

Joseph observed 7 fat years followed by 7 lean years (1). This exemplifies positive autoregressive behavior — a fluctuation above or below trend in one year implies that production will fluctuate in the next year in the same direction. Hurst, reviewing the history of the Nile

FIGURE 1  
VARIANCE BEHAVIOR OVER TIME



River, found that wet years did tend to clump with wet years, and dry years with dry (8).

Herschel suggested that grain production deviations from trend were cyclic and related to the sunspot cycles (6). Other investigators also assume cyclic behavior (3).

Ezekiel argued that poor production one year should be followed by surplus the next, and vice-versa — the cobweb hypothesis (4). This negative autoregressive behavior is based upon the effect of price incentives upon farm decisions. The current price of grain is determined by the volume of current production. This current production is influenced by last year's price. If production was low last year, prices were high. The high price influenced farmers to plant more extensively or farm more intensively, to produce a larger crop given the expectation of large profits.

One procedure for selecting a mathematical description for lagged covariance behavior over another is the comparison of empirical autocorrelation and partial autocorrelation functions with the patterns that would be generated by ideal components of processes (2). Five components of stochastic processes can be tested in this manner:

- Normal independent process (white noise),
- Autoregressive (+ or -) of lag 1, and/or 2, and/or . . . n,
- Moving average of lag 1, and/or 2, and/or . . . n,
- Cyclic, of cycle length 1, or 2, or . . . n periods,
- Mixture of autoregressive and moving average processes.

### Deviations of Residuals

Any observed and unexplained fluctuations which remain once the trend and lagged covariance effects are removed are termed residuals. If all components of the stochastic processes have been identified, the residuals should in principle be independent and randomly distributed, according to some probability rule (2). For a very short time series, such as those available for grain reserve analysis, the sample of residuals may be too small to test unambiguously for independence and consistency with a particular

Table 4 – Variables for synthetic generation

Variable name	Population parameter	Sample statistic	Definition
$I_t$			grain production (volume) in year t
t			year t
	$\mu$	u	mean production volume
	$\rho$	r	lag-one auto-correlation coefficient of production deviations
	$\sigma$	s	standard deviation of production volumes
	$\delta$	$\lambda$	a random (pseudo-random) normal deviate $\sim N(0,1)$
	$\alpha$	a	intercept of linear trend equation
	$\beta$	b	rate of change of linear trend equation
	$\vartheta$		coefficient of variation = $\sigma/\mu$
$E[ ]$			to find the expected value of items in brackets
$Var[ ]$			to find the variance of items in brackets
$E_t$			the expected production volume in year t
$D_t$			the deviation of production from the expected volume in year t
$R_t$			the random fluctuation component in production in year t

probability distribution. One procedure, followed in chapter eight, is to assume that the residuals are distributed as independent, random normal deviates.

Chapter 8 presents empirical results which suggest that a linear trend can be used to characterize a time series of historical total grains production. The work of Thomas and Fiering (10) is modified to accommodate an expected volume that increases as a linear function of time, so that appropriate synthetic futures of grain production can be generated. Their original function is presented below:

$$I_{t+1} = \mu + \rho (I_t - \mu) + \delta_{t+1} \sigma (1 - \rho^2)^{0.5} \quad (4-2)$$

The variables are defined in table 4. This expression may be thought of as three additive terms:

$$I_{t+1} = E_{t+1} + (\rho D_t) + R_{t+1} \quad (4-3)$$

with all variables as defined in table 4. Also:

$$E_{t+1} = \mu \quad (4-4)$$

$$D_t = I_t - \mu \quad (4-5)$$

$$R_{t+1} = \delta_{t+1} \sigma (1 - \rho^2)^{0.5} \quad (4-6)$$

where the new symbols have been chosen so that  $E_{t+1}$  stands for the expected value in year  $t+1$ ,  $D_t$ , the deviation from expected production in year  $t$ , and  $R_{t+1}$ , the random component of variation in year  $t+1$ .

When a linear trend is added in place of the mean value, the only change in this generating function comes in the expected volume term:

$$E_{t+1} = \alpha + \beta(t+1) \quad (4-7)$$

and the deviation from trend term:

$$D_t = I_t - (\alpha + \beta t) \quad (4-8)$$

and thus the generating function becomes:

$$I_{t+1} = \alpha + \beta(t+1) + \rho(I_t - \alpha - \beta t) + R_{t+1} \quad (4-9)$$

$$= \alpha(1-\rho) + \beta[t(1-\rho) + 1] + \rho I_t + R_{t+1} \quad (4-10)$$

When variance is constant over time, equation (4-9) can be written as:

$$I_{t+1} = \alpha + \beta(t+1) + \rho(I_t - \alpha - \beta t) + \delta_{t+1} \sigma (1-\rho^2)^{0.5} \quad (4-11)$$

Imagine that the process would not exhibit lagged covariance behavior, but rather independent normal deviations. The deviation term,  $D_t$ , would vanish because  $\rho$  equals 0, and the random shock term would reduce to:

$$R_{t+1} = \delta_{t+1} \sigma \quad (4-12)$$

because  $\rho$  equals 0. The full generating function would appear as:

$$I_{t+1} = \alpha + \beta(t+1) + \delta_{t+1} \sigma \quad (4-13)$$

A more complicated process might include a trend effect, a lagged covariance effect, and the magnitude of the variance increasing over time (in proportion to the increase in the expected volume). This last condition is known as a constant coefficient of variation. In this case, both the expected volume term,  $E_{t+1}$ , and the deviation form,  $D_t$ , would not be affected. However, the random shock term would become: 11/

$$R_t = \delta_{t+1} + (1-\rho^2)^{0.5} \alpha + \beta(t+1) \delta \quad (4-14)$$

with all variables as defined in table 4. The full generating function is now:

---

11/ This result has apparently not been published previously in the literature. However, it is a straightforward extension (7) of Thomas and Fiering's work (5; 10).

$$I_{t+1} = E_{t+1} + \rho \frac{\sigma_{t+1}}{\sigma_t} D_t + R_{t+1} \quad (4-15)$$

The two new variables,  $\sigma_t$  and  $\sigma_{t+1}$ , weight  $\rho$  by the effect of the shifting size of the standard deviations in years  $t$  and  $t+1$ .

In applications, sample estimates of these parameters are used in generating functions. Equation (4-11) would be rewritten as:

$$I_{t+1} = a + b(t+1) + r(I_t - a - bt) + \ell_{t+1} s(1-r^2)^{0.5} \quad (4-16)$$

where all variables are described in table 4.

Prior to using equation (4-16), it would be helpful to be assured that it conserves the linear trend, expected volume, the cobweb autoregressive pattern, and random normal fluctuations of production residuals. Proofs of the unbiasedness of equation (4-16) and the constant variance are shown below (9).

#### Proof of Unbiasedness of Equation (4-16)

The purpose of this proof is to show that if the model is used with many different data sets from the same population (that is, with different estimates of  $\alpha$ ,  $\beta$ ,  $\rho$ , and  $\sigma^2$ ), on average the results are correct. Assume that  $a$  and  $b$  are unbiased estimates of  $\alpha$  and  $\beta$ , respectively. By estimating the parameters by methods related to linear regression, the residuals are implicitly assumed to be independent.

The synthetic production volume in the initial year is:

$$I_0 = a + \ell_0 s \quad (4-17)$$

where  $I_0$  is production in year 0,  $\ell_0$  is the initial random deviate, and  $a$  and  $s$  are as previously defined. Note that there is no  $(1-r^2)^{0.5}$  term in the initial year. The expected volume of production in the initial year is:

$$E[I_0] = \alpha \quad (4-18)$$

Synthetic production in the next year is:

$$I_1 = a + b + r(I_0 - a) + \ell_1 s (1 - r^2)^{0.5} \quad (4-19a)$$

$$= a + b + r(a + \ell_0 s - a) + \ell_1 s (1 - r^2)^{0.5} \quad (4-19b)$$

$$= a + b + \ell_0 r s + \ell_1 s (1 - r^2)^{0.5} \quad (4-19c)$$

and the expected volume of production becomes:

$$E[I_1] = \alpha + \beta + E[\ell_0] E[rs] + 0 \quad (4-20a)$$

$$= \alpha + \beta \quad (4-20b)$$

because  $\ell_0$  and  $rs$  are independent. For the next year, the synthetic production volume would be:

$$I_2 = a + 2b + r(I_1 - a - b) + \ell_2 s (1 - r^2)^{0.5} \quad (4-21a)$$

$$= a + 2b + r[a + b + r\ell_0 s + \ell_1 s (1 - r^2)^{0.5} - a - b] + \ell_2 s (1 - r^2)^{0.5} \quad (4-21b)$$

$$= a + 2b + r^2 \ell_0 s + r \ell_1 s (1 - r^2)^{0.5} + \ell_2 s (1 - r^2)^{0.5} \quad (4-21c)$$

and the expected volume would be:

$$E[I_2] = \alpha + 2\beta + E[\ell_0] E[r^2 s] + E[\ell_1] E[rs (1 - r^2)^{0.5}] + E[\ell_2 s (1 - r^2)^{0.5}] \quad (4-22a)$$

$$= \alpha + 2\beta \quad (4-22b)$$

Then by induction:

$$E[I_t] = \alpha + \beta t \quad (4-23)$$

### Proof of Constant Variance

In this proof, unlike the previous proof, assume that  $a$ ,  $b$ ,  $r$ , and  $s$  are constants and not random variables. That can be done because we are trying to show that the model, with these constants as parameters, is homoscedastic.



The synthetic production volume and variance for the initial period are:

$$I_0 = a + \ell_0 s \quad (4-24)$$

$$\text{Var}[I_0] = s^2 \quad (4-25)$$

For the next period:

$$I_1 = a + b + r\ell_0 s + \ell_1 s (1 - r^2)^{0.5} \quad (4-26)$$

$$\text{Var}[I_1] = r^2 s^2 + s^2 (1 - r^2) \quad (4-27a)$$

$$= s^2 \quad (4-27b)$$

In the following year:

$$I_2 = a + 2b + r^2 \ell_0 s + r\ell_1 s (1 - r^2)^{0.5} + \ell_2 s (1 - r^2)^{0.5} \quad (4-28)$$

$$\text{Var}[I_2] = r^4 s^2 + r^2 s^2 (1 - r^2) + s^2 (1 - r^2) \quad (4-29a)$$

$$= r^4 s^2 + r^2 s^2 - r^4 s^2 + s^2 - s^2 r^2 \quad (4-29b)$$

$$= s^2 \quad (4-29c)$$

Then by induction:

$$\text{Var}[I_t] = s^2 \quad (4-30)$$

## SHORTFALL PATTERNS

It may not be immediately apparent how a series of generated production volumes will lead to multiyear patterns of shortfalls. Series of lean or glut years result from use of pseudo-random numbers in the generation function. The magnitude of a shortfall in any year is related to the sequence of generated random numbers. For example, if a million years of production were to be simulated, some very large normal deviates (both positive and negative) would likely be generated. When used in generated grain

production, these would lead to above- or below-average years of production (as modified by the lag-one term).

The pattern of these pseudo-random numbers produces the pattern of generated production volumes. The larger the number of years (or the number of sequences of n years) simulated, the greater the likelihood that the pattern of pseudo-random numbers will generate a long string of years of production that are above or below average.

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## V. — BUFFER STOCK SIZING METHODS

In this chapter a two- and a seven-objective linear program is formulated to optimize the size of a world, total-grains buffer stock. The constraint method of multiobjective analysis can be used to compute tradeoffs among the objectives.

The two-objective technique can generate the tradeoff curve between two objectives — maximizing food security and minimizing grain reserve size. It treats grain supply as a process which can be represented by a generating function. This function implicitly incorporates price effects, technological change, Government policies, and weather-induced fluctuations. Grain consumption is incorporated as expected demand, a deterministic quantity equal to the expected volume of production.

The seven-objective model can optimize goals of supply stabilization, price stabilization, net economic efficiency benefits, reserve authority profits, farmers' interests, and consumers' interests. The multiobjective approaches can define supply and demand for grains more flexibly than the two-objective model. Supply can be generated as an explicit function of price in the previous year and components of technological change and weather-induced fluctuations. Grain demand may be modeled as a function of price.

### FORMULATING A TWO-OBJECTIVE METHOD

Let storage at the end of period  $t$  (a decision variable) be equal to storage at the end of period  $t-1$  (a decision variable), plus whatever new production occurs in period  $t$  (given by synthetic future generation), minus whatever grain is used in period  $t$  (a decision variable):

$$S_t = S_{t-1} + I_t - U_t \quad t=1, 2, \dots, n \quad (5-1)$$

where  $S_t$  = storage at the end of period  $t$

$I_t$  = production in period  $t$

$U_t$  = grain consumed in period  $t$

Storage at the end of any period must be less than or equal to the capacity of the reserve system. This holds for all periods:

$$S_t \leq C \quad t=1, 2, \dots, n \quad (5-2)$$

where C is the storage capacity of the reserve. It is presumed that if end-of-year storages are within the buffer stock capacity, the within-year storages will not exceed that maximum size.

The level of storage at the conclusion of the final year will be defined as equal to the stock volume just prior to the beginning of the first year:

$$S_0 = S_n \quad (5-3)$$

where  $S_0$  = the level of storage just prior to the beginning of the first year

$S_n$  = the stock volume at the end of the final year

This relationship is included so that the program will neither create grain that has not been produced nor destroy grain that exists. Add the n constraints of the form (5-1). Notice that each  $S_t$  (except  $S_0$  and  $S_n$ ) appears once with a positive sign and once with a negative sign. Thus, when  $S_t$  are summed over periods  $t=1, 2, \dots, n-1$ , they cancel out. The requirement expressed as equation (5-3) permits  $S_0$  and  $S_n$  to cancel, so that the summation of production and consumption over all periods yields:

$$\sum_{t=1}^n I_t = \sum_{t=1}^n U_t \quad (5-4)$$

which states that over the design horizon the total grain released equals total production.

The expected demand level for grains in any year is assumed to be deterministic, and equal to the expected volume of production. The consumption level,  $U_t$ , can be described as a fraction of this expected demand level for any given year:

$$U_t = A_t O_t \quad t=1, 2, \dots, n \quad (5-5)$$

where  $O_t$  = volume of expected demand (or expected production); using the notation of chapter 4,  $O_t = a + bt$

$A_t$  = the fraction of  $O_t$  that is actually consumed

Substituting this expression into equation (5-1) gives:

$$S_t = S_{t-1} + I_t - A_t O_t \quad t=1, 2, \dots, n \quad (5-6)$$

Assume that the fraction of demand met through consumption will always be maintained greater than or equal to some lower bound:

$$A_t \geq B \quad t=1, 2, \dots, n \quad (5-7)$$

where  $B$  represents the smallest fraction of expected demand met through consumption over a design horizon of  $n$  years.

All decision variables take only non-negative values:

$$S_t, A_t, O_t, I_t, B, C \geq 0 \quad t=1, 2, \dots, n \quad (5-8)$$

The two objectives are to minimize the necessary storage while maximizing food security subject to constraints (5-1) to (5-8). Food security is a measure of how the stock functions to stabilize supplies over the  $n$  year design horizon. The level of food security (defined in chapter 1) is the lowest fraction of expected demand which the system can supply over a design period, which by definition is  $B$ . These stated objectives require that  $C$  be minimized and  $B$  be maximized simultaneously (table 5). All variables are as previously defined. According to custom, decision variables are included on the left-hand side of all constraints. The implicit expectation is that separate optimization of each objective will lead to different solutions; hence, the multiobjective approach.

Table 5 — The two-objective formulation

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Maximize $[B, -C]$		(5-9)
Subject to:		
$S_t - S_{t-1} + A_t O_t = I_t$	$t = 1, 2, \dots, n$	(5-10)
$S_t - C \leq 0$	$t = 1, 2, \dots, n$	(5-11)
$S_0 - S_n = 0$		(5-12)
$A_t - B \geq 0$	$t = 1, 2, \dots, n$	(5-13)
$S_t, A_t, O_t, I_t, B, C \geq 0$	$t = 1, 2, \dots, n$	(5-14)

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### THE CONSTRAINT METHOD OF SOLUTION GENERATION

Zadeh (14), Marglin (8), and Cohon (3), have shown that a linear multiobjective program can be expressed as a scalar optimization problem, with a single objective function and all other objectives incorporated in the constraint set. The constraint method of multiobjective programming generates all noninferior points by allowing the right-hand side bounds to vary over the full range of feasibility. Parametric changes in the values of these objectives incorporated as constraints generate a piecewise approximation to the noninferior set.

Imagine that a synthetic extension of the historical record of production volumes has been generated. A value of  $B$  can be designated. Now, minimize  $C$  subject to constraints (5-10) through (5-14). Parametric variation of  $B$ , while minimizing  $C$  subject to the constraint set, would generate a linear approximation to the noninferior set. This set is the tradeoff curve for the objectives of minimizing capacity and maximizing food security.

This approach, which has been called the constraint method of multiobjective programming (3, p. 134), will be referred to here as the means to generate multiobjective tradeoff surfaces. In each case, the formulation procedures are analogous to the two-objective case. Each objective is first imbedded in the constraint set. Initial values for all but one objective are set, shifting them to the right-hand side as parameters. One objective is then optimized,

subject to the constraint set. The variation of right-hand side parameter values over the full range of technical feasibility generates the tradeoff surface among the multiple objectives.

This approach implies that all objective functions can be formulated as linear equations. Some objectives are inherently linear and pose no problem (such as, "Minimize C"). Other functions, which may not be linear, could be separable and approximated by piece-wise linear functions. If such functions have the "right" shapes, the optimal solutions will represent approximations of the global optima for the original problem. <sup>12/</sup> If the shapes are not right, but the functions remain separable and amenable to piece-wise linear approximation, the solutions would at best approximate a local optimum of the original problem. Non-separable objectives cannot be used in linear multiobjective programming. See (13, pp. 550-561) for analysis of separable programming and piece-wise linear approximation.

#### A DEMAND/PRICE FUNCTION

The two-objective sizing technique did not involve prices. How can prices and benefits based upon a demand curve be developed? A relation between price and other variables is now presented.

##### A Price Function

Imagine that price in any year is related solely to physical variables, such as production, storage, consumption, and addition/release decisions for a buffer stock. A general relation between these variables may be stated as:

$$P_t = f(\bar{I}, \bar{S}, \bar{U}, \bar{Q}) \quad (5-15)$$

where  $P_t$  = price in year  $t$

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<sup>12/</sup> The right shapes for optimization are maximization of a concave function and minimization of a convex function. See (13, pp. 550-561).



$\bar{I}, \bar{S}, \bar{U}, \bar{Q}$  = vectors of production, storage, consumption, and storage decisions, respectively. Each vector consists of the value of the variable beginning with some reference year and moving to the past, such as  $I_t, I_{t-1}, I_{t-2}, \dots$

One simple price function would be the inverse of a linear demand function:

$$P_t = \gamma - \phi U_t \quad (5-16)$$

where  $\gamma$  = the intercept coefficient

$\phi$  = the rate of change coefficient

$U_t$  = the volume of grain consumed in year  $t$ ; equal to  $A_t O_t$  in equation (5-10)

Recall that the initial grain supply function allowed shifts in production over time:

$$E[I_t] = \alpha + \beta t \quad (5-17)$$

$$E[I_{t+1}] = \alpha + \beta(t+1) \quad (5-18)$$

$$E[I_{t+1}] - E[I_t] = \beta(t+1-t) = \beta \quad (5-19)$$

where  $\alpha$  = the intercept of the trend factor

$\beta$  = the rate of change in the trend factor

Analysis of grain reserves often presumes that demands and supplies do not diverge over time. For example, the expected volume of consumption should equal the expected volume of production:

$$E[I_t] = E[U_t] = O_t \quad (5-20)$$

The result of a shifting supply curve and the use of a demand curve such as equation (5-16) will result in a trend of falling prices over time. This can be seen by rewriting equation (5-16) in terms of expected demand:

$$E[P_t] = \gamma - \phi E[U_t] = \gamma - \phi O_t = \gamma - \phi(\alpha + \beta t) \quad (5-21)$$

$$= \gamma - \phi E[U_{t+1}] = \gamma - \phi[\alpha + \beta(t+1)] \quad (5-22)$$

$$E[P_{t+1}] - E[P_t] = -\phi\beta \quad (5-23)$$

The grain reserve problem specified in chapter 1 relates to supply stabilization over time; some consumption is deferred from a glut year to a lean year that may follow. The acceptance of this problem structure necessarily implies that supply and demand curves shift over time in a complementary way so that the expected volume of production always equals the expected volume of demands (1). Thus, the price of grain should be stable over time, despite shifts in both demand and supply curves.

What would such shifts entail? One can assume that the slopes of both curves remain the same and only the intercepts change. This would be consistent with parallel curve shifts. What value of  $\gamma$  implies that the demand curve will shift at the same rate as the supply curve? If  $E[P_{t+1}]$  equals  $E[P_t]$ , then by equations (5-19), (5-21), and (5-22):

$$\gamma_{t+1} - \gamma_t = \phi(O_{t+1} - O_t) = \phi\beta \quad (5-24)$$

$$\frac{\Delta\gamma}{\Delta t} = \phi\beta \quad (5-25)$$

$$\dot{\gamma} = \phi\beta \quad (5-26)$$

Integrating gives:

$$\gamma = \psi + \phi\beta t \quad (5-27)$$

where  $\psi$  is a constant of integration.

### An Illustrative Demand Function

When we substitute equation (5-27) into equation (5-16), the demand function becomes:

$$P_t = \psi + \phi\beta t - \phi U_t \quad (5-28)$$

It is possible to show that this function performs as required by checking:

$$E[P_t] = \psi + \phi\beta t - \phi(\alpha + \beta t) = \psi - \phi\alpha \quad (5-29)$$

$$E[P_{t+1}] = \psi + \phi\beta(t+1) - \phi[\alpha + \beta(t+1)] = \psi - \phi\alpha \quad (5-30)$$

## EFFICIENCY

The level of net economic efficiency benefits is the traditional cost-benefit criterion of the value of a public investment (6). One question regarding a buffer stock is whether it is an efficient investment (6; 8; 9). An analyst can see the efficiency implications of a reserve by maximizing the level of net economic efficiency benefits.

### Benefits

Gross benefits are defined as the willingness of consumers to pay — the area under the demand curve. When these benefits are expressed as an annualized, discounted dollar value in some year  $t$ , they will be termed  $GDEEB_t$ , gross discounted economic efficiency benefits in year  $t$ . An example would be to begin with the demand function (5-28) and, by integrating, develop a gross benefits function:

$$\begin{aligned} \text{Gross benefits} &= \int_0^{U_t} P_t dP_t = \int_0^{U_t} (\psi + \phi\beta t - \phi U_t) dU_t \\ \text{in year } t & \\ \text{(undiscounted)} & \end{aligned} \quad (5-31)$$

$$= \psi U_t + \phi\beta t U_t - \frac{\phi}{2} U_t^2$$

where all variables are as previously defined and the constant of integration drops out in the subtraction. If the rate of interest is  $j$  and the time series begins at  $t$  equal to 1, then discounted gross benefits are:

$$GDEEB_t = (\psi U_t + \phi\beta t U_t - \frac{\phi}{2} U_t^2) / (1+j)^{t-1} \quad (5-32)$$

Equation (5-32) is a concave function in  $U_t$  if:

$$\frac{d^2 (\text{GDEEB}_t)}{dU_t^2} \leq 0 \quad (5-33)$$

which assumes that the second derivative exists. This can be shown by checking:

$$\frac{d (\text{GDEEB}_t)}{dU_t} = \frac{\psi + \phi \beta t - \phi U_t}{(1+j)^{t-1}} \quad (5-34)$$

$$\frac{d^2 (\text{GDEEB}_t)}{dU_t^2} = \frac{-\phi}{(1+j)^{t-1}} \quad \text{and } -\phi \leq 0 \quad (5-35)$$

where  $\phi$ ,  $j$ , and  $t$  are positive by definition.

Thus, if the cost functions are either linear or convex, a net benefits function will be concave. The net benefits function can be approximated by piece-wise linear segments and solved.

The concavity result is not restricted to the example demand function; any continuous downward sloping demand function with a defined slope has a concave integral. A horizontal demand curve (where  $\phi$  equals 0) implies a linear benefits function.

### Costs of the Reserve

The reserve's costs are commitments of real resources associated with the production and storage of grain. They include expenses of growing the grain; capital costs of a grain reserve; operation and maintenance costs of the stock facility; and charges for loading or unloading grain for the reserve. <sup>13/</sup> The second through fourth expense categories will be developed here while the costs of grain production will be discussed later.

The pecuniary expenses of grain purchases by the reserve authority are not considered costs because they represent

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<sup>13/</sup> The usual approach to determination of net economic efficiency benefits is to focus upon the real incremental resources used and incremental benefits produced. Here, total resource commitments and benefits are used; hence, the inclusion of the costs of growing grain in the cost function.

only an income transfer from the reserve authority to the farmer; real resources are not used. Likewise, the income from grain sales by the reserve authority are not considered as benefits. While both expenses and revenues would be considered in the financial balance of the reserve authority, they do not affect net economic efficiency benefits.

To develop a cost function, assume that initial fixed costs are associated with construction of a buffer stock, and costs increase as the stock is scaled upward in volume:

$$\text{Capital costs} = \eta_1 + \eta_2 C \quad (5-36)$$

where  $\eta_1$  = initial fixed cost (land preparation, planning, and so on)

$\eta_2$  = cost to construct a unit of storage

$C$  = capacity

Assume that the reserve is financed through the sale of bonds which are to be repaid through annual payments on the principal plus interest, as discounted. Let the interest factor on the annualized capital costs be equal to the discount rate, both representing some social rate of time preference. Then the annual discounted costs associated with facility construction are those of equation (5-36). <sup>14/</sup>

Assume that the reserve authority pays an annual fee for the upkeep, operation, and maintenance of the facility itself. Then:

$$\begin{array}{l} \text{Annual discounted operation} \\ \text{and maintenance charge} \end{array} = \Gamma_1 / (1+j)^{t-1} \quad (5-37)$$

where  $\Gamma_1$  is the fixed fee.

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<sup>14/</sup> Imagine that a house is financed on a 20-year mortgage, with equal payments every month including some portion of interest (at rate  $j$ ) and some portion of principal. To calculate the present value of those payments, discount them by a rate of time preference, and assume an identical rate  $j$ . The real resource costs are the original capital costs of the facility. The interest rate exactly cancels the discount rate.

Let there also be a fixed fee for loading or unloading a ton of grain in the reserve. The annual costs associated with adding grain to or withdrawing it from the reserve would be the product of the transfer fee times the volume of grain moved in that year:

$$\text{Loading or unloading fee} = \Gamma_2 |S_t - S_{t-1}| \quad (5-38)$$

where  $\Gamma_2$  is the fixed transfer fee.

The absolute value term needs to be transformed before it can be used in a linear program. The absolute volume of grain moved in any year is:

$$\epsilon_t^1 - \epsilon_t^2 = S_t - S_{t-1} \quad (5-39)$$

$$Q_t = \epsilon_t^1 + \epsilon_t^2 \quad (5-40)$$

where  $\epsilon_t^1$  = the volume of grain added to the reserve in year t

$\epsilon_t^2$  = the amount of grain removed from stock in year t

$Q_t$  = the total volume of grain moved in year t

The particular formulation of equation (5-39) assures that either  $\epsilon_t^1$  or  $\epsilon_t^2$  or both will be zero in any given year where transfers are to be minimized to minimize costs. If grain is added to the reserve,  $\epsilon_t^1$  will be positive and  $\epsilon_t^2$  will be zero. If the stock is drawn down,  $\epsilon_t^1$  will be zero and  $\epsilon_t^2$  will be positive. With no net movement, both  $\epsilon_t^1$  and  $\epsilon_t^2$  will be zero. Then:

$$\begin{array}{l} \text{Annual discounted} \\ \text{transfer fees} \end{array} = \Gamma_2 Q_t / (1+j)^{t-1} \quad (5-41)$$

The sum of equations (5-36), (5-37), and (5-41) yields:

$$\begin{array}{l} \text{Total annual discounted} \\ \text{costs of the reserve} \end{array} = \eta_1 + \eta_2 C + [(\Gamma_1 + \Gamma_2 Q_t) / (1+j)^{t-1}] \quad (5-42)$$

As only  $C$  and  $Q_t$  are decision variables, this expression is a linear equation. When subtracted from a concave benefit function, it will result in a concave net benefit function.

### Grain Production Costs

One reasonable interpretation of historical world total grain production data is that its behavior is consistent with a cobweb hypothesis (chapter 8). Given such an assumption, one can develop a cost function. The cobweb hypothesis states that price in a year is influenced by the production volume of that year, but the production volume is related to the price of the previous year. Assume high production of grains in year  $t$ . This high production leads to low prices in that year. Low prices in year  $t$  will lead farmers to expect low prices in year  $t+1$ . This motivates farm decisions to maximize anticipated profits by the avoidance of costly marginal investments which would be required for large harvests. The low level of investment implies low production in year  $t+1$ ; thus, prices will rise. High prices in year  $t+1$  start the cobweb process again.

Implicit in this reasoning is an assumption that farmers will respond to increasing marginal costs of grain production by not making costly incremental investments required for large volumes of production. Such investments could either be extensive or intensive. Extensive investments involve the cultivation of marginal lands. Intensive cultivation means the addition of extra inputs to the land currently cultivated, such as more fertilizer, more or better pesticides, and increased labor.

Assume that production costs increase for every incremental unit of grain grown:

$$\begin{array}{l} \text{Cost per unit of} \\ \text{grain production} \end{array} = \Lambda_1 I_t \quad (5-43)$$

where  $\Lambda_1$  is a coefficient. The total cost of production is found by integrating the marginal cost curve. Costs of growing grain thus become:

$$\begin{array}{l} \text{Discounted cost of annual} \\ \text{grain production} \end{array} = (\Lambda_1/2) I_t^2 + \Lambda_2 / (1+j)^{t-1} \quad (5-44)$$

where  $\Lambda_2$  is a constant of integration representing the fixed costs of production. Equation (5-44) is a convex function in  $I_t$  as long as the marginal costs of production cannot decrease. Thus, the negative of equation (5-44) is concave. If equation (5-44) were subtracted from a concave benefit function, the result would be a concave curve. For any year  $t$ , let all the variables be fixed in value. The total cost curve is found by adding equations (5-42) and (5-44):

$$TDEEC_t = \eta_1 + \eta_2 C + \frac{\Gamma_1 + \Gamma_2 O_t}{(1+j)^{t-1}} + \frac{(\Lambda_1/2)I_t^2 + \Lambda_2}{(1+j)^{t-1}} \quad (5-45)$$

where  $TDEEC_t$  represents total discounted economic efficiency costs in the year  $t$ . Costs can be approximated by piece-wise linear segments.

#### Net Economic Efficiency Benefits

Net economic efficiency benefits are calculated by adding equations (5-32) and (5-45):

$$NEEB_t = GDEEB_t - TDEEC_t \quad t=1, 2, \dots, n \quad (5-46)$$

where  $NEEB_t$  is the level of net economic efficiency benefits in the year  $t$ . An objective which maximizes these benefits is:

$$\text{MAXIMIZE } Z_3 = \sum_{t=1}^n NEEB_t \quad (5-47)$$

This objective subtracts a convex and two linear curves from a concave curve; the result is concave. This objective function may be maximized without restricted basis entry; the results will be approximations of the global optima of the original problem.

This particular example, which uses function (5-28) through (5-45), may only be illustrative but it is reasonable. Future research can derive realistic benefit and cost functions.



## PROFITABILITY OF THE RESERVE

If it were true that a person could earn a certain profit by buying grain in years of plenty and selling it in years of want, there would be no reason for public interest in establishing a grain reserve. Individuals or companies would gladly enter the buffer stock business. The level of private investment in buffer stocks is related to the potential profits of operating a reserve.

Profit is defined as revenue minus cost. Revenue is derived from the sale of grain in lean years on the market. The costs include purchases of grain from the market for the stock; capital costs of reserve facility construction; operating and maintenance charges for the facility; and loading and unloading charges. Income transfers associated with sales or purchases of grain are considered in the financial balance sheet of the reserve authority.

As an example of a possible cost function, consider equations (5-36), (5-37), and (5-41) as representing the costs of facility construction, reserve operation and maintenance, and grain transfers, respectively. These costs are summarized as equation (5-42).

The revenues from grain sales and the costs of grain purchases for the buffer stock can be defined as:

$$\begin{array}{l} \text{Cash flow from grain} \\ \text{transactions in year } t \end{array} = (S_{t-1} - S_t)P_t \quad (5-48)$$

When grain is released ( $S_{t-1} > S_t$ ), this product is positive; revenues are earned on sales. When grain is added to the stock ( $S_{t-1} < S_t$ ), the product is negative; purchases of grain are costs to the reserve authority.

A problem arises in using equation (5-48) as a revenue function in a linear program. The function is a product of two decision variables, and as such is not separable. An objective function to maximize net reserve profits is:

$$RP_t = (S_{t-1} - S_t)P_t - (\eta_1 + \eta_2 C) - (\Gamma_1 + \Gamma_2 Q_t)/(1+j)^{t-1} \quad (5-49)$$

$$\text{MAXIMIZE } Z_4 = \sum_{t=1}^n RP_t \quad (5-50)$$

where  $RP_t$  are reserve profits in year  $t$ . As long as both  $P_t$  and  $S_t$  are decision variables, this objective function cannot be maximized in a linear program.

For equation (5-49) to be separable, either  $P_t$  or  $S_t$  must be redefined as a constant. One way would be if price was a function of production not consumption. As the volume of production is an exogenous variable which is an input in the sizing analysis, price levels would also be fixed. For example, equation (5-28) could be replaced by:

$$P_t = \psi + \phi\beta t - \phi I_t \quad (5-51)$$

In this case, all variables have fixed values. Equation (5-49) becomes a linear equation. Equation (5-50) can be maximized, subject to the constraint set.

The redefinition of price as a function only of production has disadvantages. It would imply that grain storage has no effect upon price. This implication may be true when a reserve is small; the stock transactions would not affect the market price. It would not be true of a much larger stock designed to stabilize prices.

## PRICE STABILITY OBJECTIVES

Chapter 1 introduced a rationale for designing a buffer stock to stabilize prices (2; 4). As a first step in formulating price stability objectives, it is useful to establish definitions of minimum price, maximum price, average price, and price deviations.

The minimum price over a reserve design horizon is less than or equal to the price of any year  $t$ :

$$P_{\min} \leq P_t \quad t=1, 2, \dots, n \quad (5-52)$$

where  $P_{\min}$  is the minimum price. The maximum price is greater than or equal to the price level in any year:

$$P_{\max} \geq P_t \quad t=1, 2, \dots, n \quad (5-53)$$

where  $P_{\max}$  is the maximum price. The mean price is the arithmetic average of all annual price levels:

$$P_{\text{ave}} = (1/n) \sum_{t=1}^n P_t \quad (5-54)$$

where  $P_{\text{ave}}$  is the average price over the reserve design lifetime. Prices in any year may deviate from the average level. The deviations in price are defined as:

$$\text{DEV1}_t - \text{DEV2}_t = P_t - P_{\text{ave}} \quad t=1, 2, \dots, n \quad (5-55)$$

$$\text{DEV1}_t, \text{DEV2}_t \geq 0 \quad t=1, 2, \dots, n \quad (5-56)$$

where  $\text{DEV1}_t$  and  $\text{DEV2}_t$  are deviations in price in the year  $t$  from the average price. Note that in a linear program, either  $\text{DEV1}_t$  or  $\text{DEV2}_t$  or both will be zero in an optimal solution (13, p. 558). If the price in a year is greater than average price,  $\text{DEV1}_t$  will be positive and  $\text{DEV2}_t$  will be zero. If the price is less than the mean,  $\text{DEV1}_t$  will be zero and  $\text{DEV2}_t$  will be positive. If an annual price level is equal to average price, both  $\text{DEV1}_t$  and  $\text{DEV2}_t$  will be equal to zero.

One price stabilization objective would be to minimize the sum of absolute price deviations, to stabilize prices around the average level over time. Large individual deviations may still occur. The formulation is:

$$\text{MINIMIZE } Z_5 = \sum_{t=1}^n \text{DEV1}_t + \text{DEV2}_t \quad (5-57)$$

Another goal might be to minimize the maximum price deviation, to reduce the most extreme price fluctuations. Small price deviations are not affected, and the sum of price deviations over time could be substantial. The maximum deviation in price is defined as:

$$\text{MAXDEV} \geq \text{DEV1}_t + \text{DEV2}_t \quad t=1, 2, \dots, n \quad (5-58)$$

where  $\text{MAXDEV}$  is the maximum deviation in price over the design lifetime of a buffer stock. The objective of minimizing this maximum deviation is:

In the multiobjective context of this chapter, equation (5-59) will turn out to be unnecessary. Later objectives will include maximizing the minimum price and minimizing the maximum price. Taken together, the set of their solutions contains the set of results from using equation (5-59).

### FARMERS' OBJECTIVES

Optimizing farmer interests is important in grain reserve analysis for two reasons. First, imagine that a policymaker is serving those farm interests. By maximizing farmers' goals, one can develop a negotiating strategy based on policy options that best suit farmers. Second, comparing the results of maximizing farmer interests versus maximizing consumer interests, price or supply stabilization, and economic efficiency, the policymaker or analyst develops some insights into who will gain and who will lose from various compromises. Two goals of farmers would be to maximize their profits and ensure a stable, high level of income over time.

Farm profits are the difference between revenues from grain sales and costs of production. Revenue is the product of market price and the volume of production. One way to formulate production costs is as the product of some fixed cost per ton of grain, times the volume of production. Profits are defined as:

$$\Pi_t = P_t I_t - [\Lambda_2 + I_t^2 + (\Lambda_1/2)] / (1+j)^{t-1} \quad (5-60)$$

where  $\Pi_t$  are profits in year  $t$ . If  $I_t$  were a decision variable, equation (5-60) would not be separable — there would be a product term of  $P_t$  and  $I_t$ . However, if  $I_t$  is determined exogenously,  $\Pi_t$  can be represented by a linear equation. Furthermore, the cost term in any year would be constant, because all variables are defined exogenously.

When total farmer costs can be considered constant, maximization of revenues yields the same optimal result as

the maximization of profits. A general definition of profits is:

$$\Pi = R - K \quad (5-61)$$

where  $\Pi$  equals profits,  $R$  equals revenue, and  $K$  equals cost (a constant by assumption). Profits will be maximized when:

$$d\Pi / dQ = 0 \quad (5-62)$$

Now

$$d\Pi / dQ = dR / dQ - dK / dQ = dR / dQ \quad (5-63)$$

because  $dK/dQ$  is 0, as  $K$  is a constant. Thus, when profits reach their maximum level ( $d\Pi/dQ=0$ ), the level of revenues also reach their maximum ( $dR/dQ=0$ ). In short, if a goal is to maximize the sum of the net discounted farm profits over a time period, it is sufficient under conditions of constant costs to maximize the sum of the net discounted revenues over that period. Discounted farm revenue in any year  $t$  is:

$$FP_t = P I_t / (1+j)^{t-1} \quad (5-64)$$

where  $FP_t$  is discounted farm revenue in year  $t$ .

The formulation of the objective which maximizes the net discounted value of farm revenues over time is:

$$\text{MAXIMIZE } Z_7 = \sum_{t=1}^n FP_t = \sum_{t=1}^n P_t I_t / (1+j)^{t-1} \quad (5-65)$$

If equation (5-28) is the price function, then by substitution:

$$\sum_{t=1}^n P_t I_t / (1+j)^{t-1} = \sum_{t=1}^n (\psi + \phi\beta t - \phi U_t) I_t / (1+j)^{t-1} \quad (5-66)$$

Equation (5-66) can be maximized with linear programming; the result will be the optimum of the original problem.

Farmers may wish to ensure a stable flow of income over time. One way to achieve this would be to design a reserve to keep a high floor on the bottom fluctuations of price. An objective of maximizing the minimum price would tend to stabilize incomes by increasing as far as possible the minimum price farmers receive for their grain. Equation (5-52) defined the minimum price over the life of a grain reserve. The objective of maximizing minimum price is formulated as:

$$\text{MAXIMIZE } Z_8 = P_{\min} \quad (5-67)$$

### CONSUMERS' OBJECTIVES

The optimization of objectives which reflect consumer interests makes sense either for serving those consumer goals or as part of a multiobjective analysis. Consumers, whether as individuals, interest groups, or importing nations, would be affected by any grain reserve policy. One of their goals would be to maximize consumer surplus, the difference between what the consumer is willing to pay and what the market forces him to pay. A second goal might be to ensure a stable, low level of expenditures over time.

Consumers' willingness to pay is described by the demand curve, the function which relates the quantity that consumers will demand as a function of price:

$$Q_D = f^{-1}(P) \quad (5-68)$$

where  $Q_D$  equals the quantity demanded by consumers, and  $f^{-1}$  equals a symbol denoting an inverse of the price function. Aggregate consumer willingness to pay is the area under the demand curve, defined in equations (5-31) and (5-32) as the gross economic efficiency benefits.

What the consumer does pay for grain is the product of the amount consumed by the price:

$$\text{Consumer expenditure} = P_t U_t \quad (5-69)$$

One problem with equation (5-69) is that consumer expenditure is a product of two decision variables, and hence is not separable. Substitution of the inverse demand function, equation (5-28), into equation (5-69) yields:

$$\begin{aligned} P_t U_t &= (\psi + \phi\beta t - \phi U_t) U_t \\ &= \psi U_t + \phi\beta t U_t - \phi U_t^2 \end{aligned} \quad (5-70)$$

which is separable. When equation (5-70) is discounted, at a rate of interest  $j$  over a time series beginning at  $t$  equals 1, it follows that:

$$P_t U_t = (\psi U_t + \phi\beta t U_t - \phi U_t^2) / (1+j)^{t-1} \quad (5-71)$$

Consumer surplus in any year  $t$  is the difference between the area under the demand curve and the product of the volume of grain consumed times the price. This is calculated by subtracting equation (5-71) from equation (5-32):

$$CS_t = (\psi U_t + \phi\beta t U_t - \frac{\phi}{2} U_t^2 - \psi U_t - \phi\beta t U_t + \phi U_t^2) / (1+j)^{t-1} \quad (5-72)$$

when common terms cancel:

$$CS_t = \frac{\phi}{2} U_t^2 / (1+j)^{t-1} \quad (5-73)$$

Consumer surplus is thus a convex function in consumption ( $U_t$ ) terms. The objective — maximize consumer surplus over the grain reserve investment life — would be formulated as:

$$\text{MAXIMIZE } Z_9 = \sum_{t=1}^n CS_t \quad (5-74)$$

Equation (5-74) is convex, as it is the sum of convex functions of the form of equation (5-73). The right shape for linear programming is to maximize a concave function subject to linear constraints. Thus, to optimize objective consumer surplus requires separable programming — piecewise linear approximations of the nonlinear objective function used with restricted basis entry. The solution cannot be demonstrated

to be a global maximum, although it will be a feasible local optimum.

Consumers may wish to ensure a stable, low level of expenditures for grain over the lifetime of the buffer stock. A reserve which enforces a low price ceiling would damp upward fluctuations of price. Such an objective differs from the control of price, because no price is set. Equation (5-53) defined the maximum price in a design period for a buffer stock. The objective of minimizing this maximum price, formulated as equation (5-75), can be optimized without modification in a multiobjective linear program:

$$\text{MINIMIZE } Z_{10} = P_{\max} \quad (5-75)$$

### COMPUTATIONAL AND DISPLAY ISSUES

Ten different objectives for grain reserve sizing have been examined. Nine of the ten can be used in a multiobjective linear programming context, and the most useful seven are in table 6. The complete constraint set is shown in table 7.

Table 6 – Grain reserve objectives

1. Minimize capacity: $\text{Min } C$	(5-77)
2. Maximize food security: $\text{Max } B$	(5-78)
3. Maximize net economic efficiency benefits: $\text{Min } \sum_{t=1}^n \text{NEEB}_t$	(5-79)
4. Minimize price deviations: $\text{Min } \sum_{t=1}^n \text{DEV } 1_t + \text{DEV } 2_t$	(5-80)
5. Maximize farmer revenues (profits): $\text{Max } \sum_{t=1}^n \text{FP}_t$	(5-81)
6. Maximize minimum price: $\text{Max } (P_{\min})$	(5-82)
7. Minimize maximum price: $\text{Min } (P_{\max})$	(5-83)



Table 7 – The constraint set

$S_t - S_{t-1} + A_t O_t = I_t$	$t = 1, 2, \dots, n$	(5–6)
$S_t - C \leq 0$	$t = 1, 2, \dots, n$	(5–2)
$A_t - B \geq 0$	$t = 1, 2, \dots, n$	(5–7)
$S_0 - S_n = 0$		(5–3)
$S_t, A_t, O_t, I_t, C, B \geq 0$	$t = 1, 2, \dots, n$	(5–8)
$P_t - \phi A_t O_t = \psi + \phi \beta t$	$t = 1, 2, \dots, n$	(5–28 and substitution)
$NEEB_t - GDEEB_t + TDEEC_t = 0$	$t = 1, 2, \dots, n$	(5–32), (5–45), (5–46)
$P_t - P_{\min} \geq 0$	$t = 1, 2, \dots, n$	(5–52)
$P_t - P_{\max} \leq 0$	$t = 1, 2, \dots, n$	(5–53)
$(1/n) \sum_{t=1}^n P_t - P_{ave} = 0$		(5–54)
$DEV1_t - DEV2_t - P_t + P_{ave} = 0$	$t = 1, 2, \dots, n$	(5–55)
$FP_t - P_t I_t / (1 + j)^{t-1} = 0$	$t = 1, 2, \dots, n$	(5–64)
$P_t, GDEEB_t, TDEEC_t, P_{\min}, P_{\max},$ $DEV1_t, DEV2_t, P_{ave} \geq 0$	$t = 1, 2, \dots, n$	(5–76)

The use of so many objectives poses computational problems:

- . How will the problem be solved?
- . How much will it cost?
- . If results are obtained for  $n$  greater than 3, how can a  $n$ -dimensional tradeoff surface be displayed?

The constraint method of multiobjective linear programming can generate the tradeoff surface among any subset of those objectives (3, p. 134). For example, let the price stabilization objective, equation (5-80), be placed as the objective function. Let initial values be set for each of the elements of the set  $\{C, B, \Sigma NEEB_t, \Sigma FP_t, P_{min}, \text{ and } P_{max}\}$ . Place these initial values in the right-hand sides of equations (5-2), (5-7), (5-46), (5-64), (5-52), and (5-53), respectively. Optimize objective four subject to the constraint set and the designated right-hand side values of the other six objectives. Let the right-hand sides be perturbed parametrically until the full range of feasibility is explored. Optimization of the program with the parametric changes will trace out the seven-dimensional tradeoff surface representing five goals: supply stabilization; price stabilization; farmer interests; consumer interests; and net economic benefits. It might be sufficient to consider only objectives 2, 3, 4, 5, and 7.

The number of linear programming solutions required to trace out the multidimensional tradeoff surface is a function of the number of objectives and the number of values for each objective deemed adequate to approximate the surface contour. Let  $T$  values of each objective represent the number of solutions desired by the analyst to describe the tradeoff surface. If there are  $\Omega$  objectives, the number of necessary solutions is (3, p. 139):

$$\text{Necessary solutions} = T^{\Omega-1} \quad (5-84)$$

For a five-objective problem, when 10 values of each objective are desirable, the number of necessary solutions is 10 to the fourth power, or 10,000. If only six values of each objective are sufficient, then 1,296 solutions are sufficient. If a sixth objective were added (while main-

taining 10 values per objective), the number of solutions would jump to 100,000.

Such computational effort can be costly. The cost per linear programming solution of the two-objective formulation averaged less than 25 cents. Even at the rate of 25 cents a solution, the cost of 10,000 is \$2,500. However, the augmented five-objective problem would surely be more expensive because it has a larger constraint set and more variables.

A final consideration is how to display a multidimensional tradeoff surface. The conventional approach would be to make comparisons between any pair of objectives. Tradeoff curves in two dimensions would be thus constructed for every possible set of two objectives. For a five-objective problem, the number of tradeoff curves is the number of combinations of five things taken two at a time:

$$\text{Number of tradeoff curves} = \binom{5}{2} = \frac{5!}{2!(5-2)!} = 10 \quad (5-85)$$

The display approach has two disadvantages — it is tedious and perhaps myopic. The policymaker sees only two-dimensional slices from a five-dimensional surface. So much detail could make important implications of decisions difficult to perceive. The subject of how to display a higher dimensional tradeoff surface (more than three objectives) is a topic of active research (11).

## EQUILIBRIUM RESERVE SIZING

All of the previous grain reserve sizing techniques have treated the processes of supply and demand for grains as separate phenomena. Supply has thus far been treated as an exogenous variable. Historical production is analyzed to develop a function which can generate series of production statistics with identical expected volume, variance, and lagged covariance behavior to the original data. Synthetic futures are generated, and fed in as exogenous fixed values into the sizing algorithms.

The generating functions can include an autoregressive term that attributes a negative serial correlation to productions. Such a term implies that high production fluctuations are more likely to follow low fluctuations than under random, independent conditions. This autoregressive

term does yield the same type of behavior as that implied by the cobweb hypothesis — current price is determined by current production, but current production is a function of last year's price (5). The difference is that the generating function is a separate process, without the interaction of a price system or demands for grain. A different model of supply might be more specific about potential explanatory variables for the cobweb behavior of production fluctuations. Let us now develop an equilibrium model of grain reserve sizing in which supply is defined as a process endogenous to the sizing analysis. It will be affected by technological change, random weather fluctuations, and behavior induced by lagged price.

The demand side of previous formulations used two notions. The expected volume of demand was deterministic and equal to the expected volume of production. Grain consumption was the quantity actually used in a year; it equaled the production as modified by storage or release behavior. Another view is when consumption demand is a function of price.

A cobweb model might state the three elements explicitly as equations (5-86) through (5-88):

$$I_t = f(P_{t-1}) \quad t=1, 2, \dots, n \quad (5-86)$$

$$U_t = f(P_t) \quad t=1, 2, \dots, n \quad (5-87)$$

$$I_t = U_t \quad t=1, 2, \dots, n \quad (5-88)$$

where  $I_t$  = production in year  $t$

$f( )$  = is a function of the variables in parentheses

$U_t$  = consumption in year  $t$

$P_t$  = average annual price in year  $t$

In a descriptive cobweb model, equilibrium is achieved through market clearance, not imposed public objectives. Assume that specific functional forms for equations (5-86) through (5-88) can be estimated from data and that an initial value of price ( $P_0$ ) is designated. The time path of both production and consumption for years  $t=1, \dots, n$  is determined by those equations.

A grain reserve sizing technique, on the other hand, is not intended to be descriptive but prescriptive. An equilibrium model of grain reserve sizing would relate demand to supplies via price-mediated market activity. Equilibrium is to be imposed through market allocations that result from prices but involve Government storage or withdrawal behavior which responds to explicit objectives. Price is no longer the residual effect of equilibrating supplies and demands. The resulting equilibrium can be affected by price as a decision variable.

A reserve-oriented optimization version might replace equation (5-88) by:

$$I_t = U_t + S_t - S_{t-1} \quad t = 1, 2, \dots, n \quad (5-89)$$

where  $S_t$  is storage in year  $t$ . Values of  $S_t$  could be determined by some reserve sizing technique which achieves some stated objective(s).

### Grain Supply

Assume the level of grain production in any year is a function of technological change (a trend effect); a cobweb-type lagged price effect; and a random fluctuation due to weather or other causes. Assume the variance of crop production is stable over time, the presumption of homoscedasticity. Let the trend effect be defined as a linear increase in expected production over time:

$$E_t = \alpha + \beta t \quad t = 1, 2, \dots, n \quad (5-90)$$

where  $E_t$  = expected volume of production in year  $t$

$\alpha$  = intercept of the linear trend equation

$\beta$  = rate of change in the linear trend equation

$t$  = the year  $t$

Let lagged price be assumed to affect production, which leads to cobweb-type behavior of production over time. High production in year  $t$  leads to low prices in that year. Low prices in year  $t$  imply farmers' expectations of low prices in the coming year,  $t+1$ . The low

price expectations motivate farmers to maximize expected profits by avoiding costly marginal investments which would be required for large harvests. This lower level of investment leads to low production in year  $t+1$ . Low production in year  $t+1$  implies a high price in that year, and the process continues. The cobweb effect can be formalized as:

$$L_t = \rho_{IP}^1 \left( \frac{\sigma_I}{\sigma_P} \right) (P_{t-1} - P_{ex}) \quad t=1, 2, \dots, n \quad (5-91)$$

where  $L_t$  = the lag-one effect

$\rho_{IP}^1$  = the correlation coefficient between price fluctuations in one year and production volumes in the next year

$\sigma_I$  = the standard deviation of production volumes

$\sigma_P$  = the standard deviation of prices

$P_{ex}$  = the expected value of the random variable price (developed formally in the next section).

The random shock effect term would be:

$$R_t = \delta_t \sigma_I [1 - (\rho_{IP}^1)^2]^{0.5} \quad t=1, 2, \dots, n \quad (5-92)$$

where  $R_t$  = the random shock term in year  $t$

$\delta_t$  = a random deviate selected from a standard normal distribution(7),  $\delta \sim N(0,1)$

The generating function of production is now:

$$I_t = E_t + L_t + R_t \quad t=1, 2, \dots, n \quad (5-93)$$

The principal difference between equation (5-93) and earlier generating functions is that this function need not be treated as exogenous to the grain reserve sizing methodology.

Let random normal deviates be provided as input data. <sup>15/</sup> Equation (5-93) could then be incorporated in a reserve sizing formulation as both a synthetic generating function and a linear program constraint. Production becomes a process affected both by natural and market forces. The cobweb effect of lagged price fluctuations is incorporated explicitly.

### Demand for Grains

Grain demand should be related to price because market equilibrium is achieved by price-mediated clearance of supplies and demands. In a grain reserve model, demand may also be related to production. Indeed, the yearly shift in the demand curve should compensate the yearly shift in the supply curve so that the expected volume of demand always equals the expected volume of production:

$$E[I_t] = E[U_t] = O_t = \alpha + \beta t \quad t=1, 2, \dots, n \quad (5-94)$$

where  $O_t$  is the expected volume of demand and production. This implies that the expected real (uninflated) price of grains should remain constant over time. Previously a demand function with these properties was presented:

$$P_t = \psi + \phi\beta t - \phi U_t \quad t=1, 2, \dots, n \quad (5-28)$$

where  $\psi$ ,  $\phi$ , and  $\beta$  are constants. The  $\beta$  is the same  $\beta$  associated with equation (5-90). To calculate expected price, substitute  $O_t$  for  $U_t$ :

$$E[P_t] = \psi + \phi\beta t - \phi O_t \quad (5-95)$$

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<sup>15/</sup> This assumes that the residual deviations of production (after the removal of the trend and lagged covariance portions of the variability) are normally distributed. This assumption is not necessary for analysis. Another distribution could be assumed or empirical data could be statistically evaluated.

## The Equilibrium Cobweb

The deterministic cobweb system of equations (5-86), (5-87), and (5-88) contained a supply function, a demand function, and an equilibrium condition. In the reserve-oriented approach, there is a supply function (5-93) and a demand function (5-28). Equation (5-89) can act as an equilibrium condition to close the cobweb and clear the market. Recall that equation (5-89) determined the behavior over time of the cobweb system, given initial conditions. Equation (5-89) also allows both the cobweb behavior and the market clearance to respond to normative commands of objectives. Once storage decisions have been made in response to an objective, both market and cobweb behavior are determined.

Equations (5-93), (5-28), and (5-89) can be added to the constraint set described in table 7 and used in a grain reserve sizing analysis with objectives such as those in table 6. As  $I_t$  is no longer a fixed parameter, but now an unknown variable determined endogenously through a supply function, some of these objectives are no longer separable. Objective function (5-81) is no longer separable because it is calculated as the product of two decision variables,  $I_t$  and  $P_t$  (see equation (5-64)). In the initial two-objective model of table 5, only one boundary condition was required — an initial or final level of storage. For this equilibrium model, the initial price ( $P_0$ ) and the initial storage volume ( $S_0$ ) must be fixed.

The comparative advantage of an equilibrium formulation is that it incorporates explicitly a market structure in the grain reserve analysis. Consumption and production are no longer separate processes. Now demands are affected by supply fluctuations. Grain supply is related to price as well as technological change and weather-induced fluctuations. A market operates to balance supply and demand, in response to grain reserve actions which reflect decisions to achieve some mix of public objectives.

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## VI. — RELIABILITY ANALYSIS

Few researchers have evaluated the potential for a series of back-to-back gluts or shortfalls. Procedures are now developed to estimate the likelihood of sequential good or lean years.

It is important to determine how well a reserve design based on the past will perform in an uncertain future. The two-objective technique developed in chapter 5 sizes a grain reserve to minimize capacity and maximize food security. The model calculates the smallest stock size which could release sufficient grain to stabilize the volume of grain available for consumption over one investment lifetime. What is the value of these results? How effectively would the reserve perform through another sequence of years; would it be a reliable buffer for food stocks?

Reliability is defined as the probability that a designated reserve size will perform at least as well as a stated level of food security. As a probability measure, reliability can take on any value between 0.0 and 1.0. Imagine that design capacity C performs at food security level B, based on historical production data. If the future is assumed to be identical to the past, the reliability of C is 1.0. What happens if the future is not identical to the past? Assume that future production will mimic history only to the extent of repeating the expected volume, variance and lagged covariance statistics. This chapter develops a methodology to evaluate explicitly performance reliability for synthetic futures.

One issue is how to estimate reliability, a population parameter, based on a sample of stochastic futures. A stochastic vision of the future implies that the "true" underlying reliability can never be determined. A capacity C may perform at least as well as food security level B in a large number of samples of investment lifetimes. But what about its performance in the larger number of possible production sequences which were not observed?

A second issue is how to recognize when results have a high probability of being highly reliable. This issue is difficult to describe in words, because there are two levels of probabilistic reasoning related to "a level of reliability" and "the probability of performing reliably."

A third issue is how to maximize result reliability while incurring the smallest possible computational burden.

What is the relation between added certainty from additional analysis and incremental costs?

## BACKGROUND

To begin, let us define a "worst shortfall." Allow that it is possible to calculate a trend (expected volume) of grain production from the historical record. If in a given year the actual production falls below trend, the difference is a shortfall. Let the "worst" shortfall of any time series be defined as the largest aggregate sum of shortfalls from any number of sequential years when production falls below trend. This "worst" shortfall may be one very lean year or the sum of back-to-back shortfalls.

For example, table 8 shows the "worst" shortfall during 1960-74. Column two lists total world grain production (all food and feed grains) according to an USDA data series (9). Column three lists trend production volumes estimated through the use of a linear regression equation. Column four shows the annual shortage or surplus, the difference between columns two and three. The "worst shortfall" is 77 million metric tons during the period 1963 through 1966.

Assume we can develop a function capable of generating many time series of production volumes that mimic the statistical behavior of the historical record. Further, allow that production during some predefined future period of  $q$  years will continue in the statistical pattern of the historical record. We can then estimate the magnitude of sequential grain production shortfalls.

Let  $n$  separate simulations, each of length  $q$  years, be generated by the function. For each of the time series, a "worst shortfall" can be calculated. Let  $Y$  be a random variable representing the amount of "worst shortfall" for a simulation (upper case letters denote random variables and lower case letters denote their realizations). For  $n$  simulations, let:

$$Y_{(1)}, Y_{(2)}, \dots, Y_{(m)}, \dots, Y_{(n)} \quad (6-1)$$

be random variables representing the order statistics of the sample  $Y$ , where  $Y_{(1)}$  represents the smallest of the "worst" shortfalls in  $n$  simulations,  $Y_{(2)}$  the second

Table 8 — Calculation of "worst" shortfall in a time series

Year	Actual production *	Estimated production (million metric tons)	Difference
1960	888.6	845.2	+ 43.4
1961	862.4	874.9	— 12.5
1962	908.7	904.6	+ 4.1
1963	910.7	934.3	— 23.6
1964	956.1	964.0	— 7.9
1965	952.2	993.6	— 41.4
1966	1,019.2	1,023.3	— 4.1
1967	1,060.8	1,053.0	+ 7.8
1968	1,103.3	2,082.7	+ 20.6
1969	1,118.0	1,112.4	+ 5.6
1970	1,122.8	1,142.1	— 19.6
1971	1,211.6	1,171.7	+ 39.9
1972	1,175.8	1,201.4	— 25.6
1973	1,282.8	1,231.1	+ 51.7
1974	1,222.1	1,260.8	— 38.7

\* Data from (9).

smallest,  $Y_{(m)}$  the  $m^{\text{th}}$  smallest, and  $Y_{(n)}$  the largest "worst" shortfall in  $n$  simulations.

For example, the left-hand side of figure 5 in chapter 8 illustrates world total grain production for 1960-74 and two synthetic production series for the same period. View this set as a group of  $n$  equals 3 simulations of  $q$  equals

15 years. Table 9 lists the "worst" shortfalls for these series and the associated number of back-to-back lean years. In this case,  $Y(1)$  equals 46.5 MMT,  $Y(2)$  is 68.8 MMT, and  $Y(n)$  equals 77.0 MMT.

Table 9 - A set of three "worst" shortfalls, 1960-74

Series	Worst shortfall	Consecutive lean years represented	Order statistic
	(Million metric tons)	(number)	
Historical	77.0	4	$Y_{(n)}$
Synthetic no. 1	46.5	2	$Y_{(1)}$
Synthetic no. 2	68.8	1	$Y_{(2)}$

#### Derivation of a Cumulative Distribution Function

How likely is it that a shortfall in the next  $q$  years will be larger than the  $m^{\text{th}}$  of  $n$  simulated "worst" shortfalls? Let  $G_{(m)}$  be defined as the probability that the  $m^{\text{th}}$  simulated shortfall will not be exceeded by a real shortfall over the next  $q$  years; that is:

$$G_{(m)} = P \{ Y < Y_{(m)} \} \quad (6-2)$$

$$= F_y(Y_m)$$

where  $F_y(.)$  is the unknown cumulative distribution function of  $Y$ . Note that because  $Y_{(m)}$  is a random variable,  $G_{(m)}$  is also a random variable. Thus we cannot determine the value of  $G_{(m)}$ . However, we can evaluate the probability distribution of  $G_{(m)}$  and compute an expected value and confidence limits. This can be done without assuming the form of the underlying distribution of  $Y$  (7).

Let  $Y$  be a random variable representing the amount of the largest aggregate sum of shortfalls for a simulation. Let the unknown probability distribution function of  $Y$  be designated as  $f_y(.)$ . Also let:

$$G = F_Y(Y) \quad (6-3)$$

Note that  $G$  is a random variable, distributed uniformly on the interval  $[0,1]$  (4). Thus:

$$F_G(g) = g \quad (6-4)$$

$$f_G(g) = 1 \quad (6-5)$$

Consider a random sample of size  $n$  on  $Y$ . Let  $Y_{(m)}$  be the  $m$ th order statistic of the sample. Also let:

$$G_{(m)} = F_Y(Y_{(m)}) \quad \text{for } m=1, 2, \dots, n \quad (6-6)$$

Note that  $G_{(m)}$  is a random variable. Since  $F_Y(\cdot)$  increases monotonically:

$$Y_{(1)} \leq Y_{(2)} \leq \dots Y_{(m)} \leq \dots Y_{(n)} \quad (6-7)$$

then:

$$G_{(1)} \leq G_{(2)} \leq \dots G_{(m)} \leq \dots G_{(n)} \quad (6-8)$$

Thus  $G_{(m)}$  is the  $m$ th order statistic on a random sample of size  $n$  on the random variable  $G$ . The probability distribution function of  $G_{(m)}$  is given by:

$$f_{G_{(m)}}(g) = \binom{n}{m} m [F_G(g)]^{m-1} [1 - F_G(g)]^{n-m} f_G(g) \quad (6-9)$$

where  $n$  is the number of simulations,  $m$  is the rank of the order statistic, and  $\binom{n}{m}$  is the binomial coefficient,  $n!/m!(n-m)!$  (4, p. 253). Substituting for  $F_G(g)$  and  $f_G(g)$  in equations (6-4) and (6-5), the result is:

$$f_{G_{(m)}}(g) = \binom{n}{m} m g^{m-1} (1-g)^{n-m} \quad (6-10)$$

which is the expression for the probability distribution of  $G_{(m)}$  we wished to derive.

### Methodology Development

Based on equation (6-10), the expected probability that no shortfall over the coming  $q$  years will exceed the  $m^{\text{th}}$  worst shortfall is given by:

$$\begin{aligned} E\{\Pr[Y \leq Y_{(m)}]\} &= E[G_{(m)}] & (6-11) \\ &= \int_0^1 f_{G_{(m)}}(g) dg \\ &= \binom{n}{m} m \int_0^1 g^{m-1} (1-g)^{n-m} dg \\ &= \frac{m}{n+1} \end{aligned}$$

If the intent is that no massive sequence of back-to-back lean years exceed the largest "worst shortfall" on record,  $m$  should be set equal to  $n$ , giving:

$$E\{\Pr[Y \leq Y_{(n)}]\} = \frac{n}{n+1} \quad (6-12)$$

One can also compute confidence limits for  $G_{(m)}$ . Let  $g_1$  and  $g_2$  be either two estimates of  $G_{(m)}$  or two arbitrary bounds. Based upon equation (6-10):

$$\begin{aligned} \Pr[g_1 \leq G_{(m)} \leq g_2] &= \int_{g_1}^{g_2} f_{G_{(m)}}(g) dg & (6-13) \\ &= \binom{n}{m} m \int_{g_1}^{g_2} g^{m-1} (1-g)^{n-m} dg \end{aligned}$$

In general, for given values of  $n$  and  $m$ , evaluation of equation (6-13) requires  $(n-m)$  successive integrations by parts. For determining the probability that the largest "worst" shortfall will not be exceeded during the next  $q$  years, the value of  $m$  is set equal to  $n$ . The probability that  $G_{(n)}$  falls within the bounds  $g_1$  and  $g_2$  is:



$$\begin{aligned}
 \Pr[g_1 \leq G_{(n)} \leq g_2] &= \binom{n}{n} n \int_{g_1}^{g_2} g^{n-1} dg \\
 &= n \left[ \frac{1}{n} g^n \right] \Big|_{g_1}^{g_2} \\
 &= g_2^n - g_1^n
 \end{aligned}
 \tag{6-14}$$

Similarly, the probability that  $G_{(n)}$  is greater than or equal to some estimate  $g_1$  is:

$$\Pr\{G_{(n)} \geq g_1\} = 1 - g_1^n \tag{6-15}$$

and the probability that  $G_{(n)}$  is less than or equal to some estimate  $g_2$  is given by:

$$\Pr\{G_{(n)} \leq g_2\} = g_2^n \tag{6-16}$$

Without equations (6-12), (6-15), and (6-16), we cannot easily estimate the likelihood that a grain shortfall during the next  $q$  years will exceed the largest "worst" shortfall observed in  $n$  simulations of future production. Given these results, we can calculate the expected values of expressions that this parameter will fall below, between, or above any arbitrary (or purposeful) limit. Furthermore, these results hold regardless of the expected volume, variance, and lagged covariance behavior of the original data series. These results are independent of the probability distribution of the fluctuations of production, and they are not affected by the length of the planning time horizon. This nonparametric approach could be used with any simulations analysis to evaluate the probability that a result could be improved through additional simulation runs.

This nonparametric procedure has advantages over the confidence interval estimation approach typically employed to assess the likelihood of severe grain shortfalls (1-3; 5-6; 8). In the interval estimation approach, the analyst assumes that the random variable "grain shortfall in a given year" is distributed normally. The analyst computes estimates of a confidence interval representing upper and

lower bounds upon the magnitude of grain shortfalls or surpluses for individual years (for examples, see 8, p. 17).

The order statistics method does not require an assumption regarding the probability distribution of the fluctuations of yields or production about the trend level. Moreover, it can evaluate directly for back-to-back lean years. As illustrated in equations (6-12), (6-15), and (6-16), the value of the parameter and the probability of its location are determined only by the number of simulations of future production and the value of the bounds. As the analyst controls both these variables, it is possible to assure with any desired degree of reliability that no future shortfall will exceed the largest one observed in the simulation. The practical limit of reliability is set by the computer budget of the investigator.

#### APPLICATION OF THE ORDER STATISTICS METHOD

Let us now use the order statistics method to determine the numerical equivalent of  $G_{(n)}$  and the certainty of its value.

Suppose that nine synthetic futures have been analyzed with the two-objective sizing method of chapter 5. Assume that a reserve capacity (C) achieves complete food security (B). How reliably does C stabilize food security through the worst sequence of shortfalls that could occur based on existing information?

The expected value of the reserve reliability is:

$$E[G_{(n)}] = n/(n+1) = 9/10 = 0.9 \quad (6-17)$$

Let two estimates of reliability be selected arbitrarily as  $g_1 = 0.8$  and  $g_2 = 0.95$ . The probability that true reliability is between these estimates is:

$$\begin{aligned} \Pr [0.8 \leq G_{(n)} \leq 0.95] &= g_2^n - g_1^n = (0.95)^9 - (0.8)^9 \quad (6-18) \\ &= 0.630 - 0.134 \\ &= 0.486 \end{aligned}$$

The probability that the true reliability is greater than the second estimate, 0.95, is:

$$\Pr[G_{(n)} \geq 0.95] = 1 - g_2^n = 1 - (0.95)^9 = 0.370 \quad (6-19)$$

In chapter 8 empirical results are developed from a sample of 34 simulations of production for 1975-2000. Based on  $n$  equal to 34, the expected value of the reliability of any stock to stabilize supplies would be:

$$E[G_{(n)}] = 34/35 = 0.971 \quad (6-20)$$

Let three bounds on  $G_{(n)}$  be arbitrarily defined as  $g_1 = 0.9$ ,  $g_2 = 0.95$ , and  $g_3 = 0.99$ . The probability that  $G_{(n)}$  exceeds these bounds can be calculated with equation (6-15):

$$\Pr[G_{(n)} \geq 0.90] = 1 - (0.9)^{34} = 0.972 \quad (6-21)$$

$$\Pr[G_{(n)} \geq 0.95] = 1 - (0.95)^{34} = 0.825 \quad (6-22)$$

$$\Pr[G_{(n)} \geq 0.99] = 1 - (0.99)^{34} = 0.289 \quad (6-23)$$

These results illustrate that it is not easy to have both a high certainty of  $G_{(n)}$  and a high value of  $G_{(n)}$ . As the likelihood increases that no future shortfall will exceed the worst shortfall observed, the certainty of that likelihood decreases. The obverse question, examined next, is "how many simulations should be run in order to achieve a pre-defined level of reliability?"

The number of simulations of future production directly affects the values of the probability that a future shortfall will exceed the largest shortfall found through a simulation. Imagine that 1,000 scenarios are run, the worst shortfall found for each, and the largest of those 1,000 identified. The expected value of the probability that no shortfall would exceed the largest one found through simulation would be:

$$E[G_{(n)}] = 1,000/1,001 = 0.999 \quad (6-24)$$

which exceeds the value of 0.971 in (6-20) resulting from 34 simulations. While 1,000 simulations are more likely to uncover a larger shortfall from back-to-back lean years, they also represent a thirtyfold increase in the computational burden. Indeed, the analyst can use equation (6-15) to determine how many simulations should be evaluated to achieve a predetermined performance reliability level. Let (6-15) be rewritten to include general lower bound:

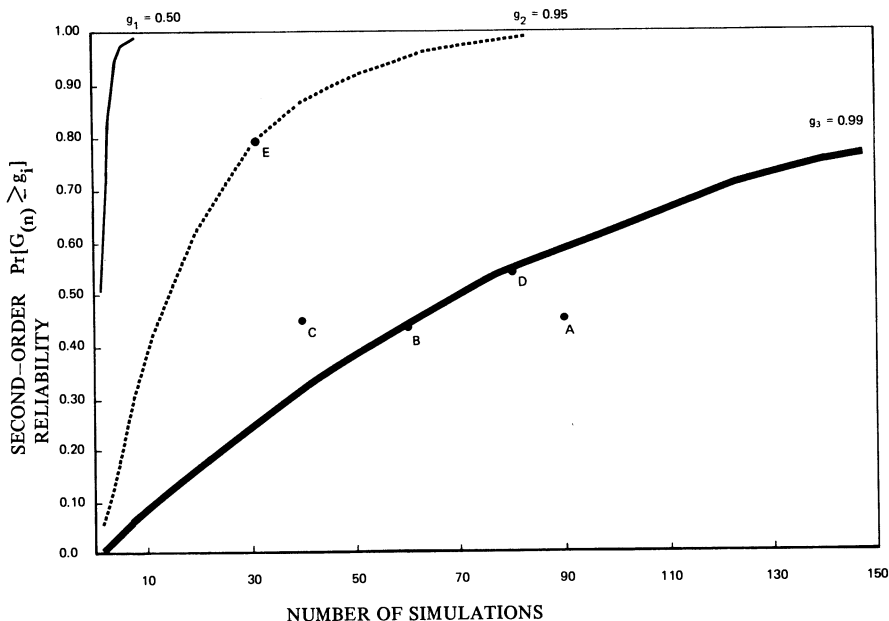
$$\Pr[G_{(n)} \geq g_i] = 1 - g_i^n \quad (6-25)$$

where  $n$  is the number of simulations and  $g_i$  is the  $i$ th lower bound on the level of probability that no cumulative sequence of lean years will exceed the largest worst shortfall found through simulation.

The tradeoff between certainty and computational burden is illustrated by the following example. Let three lower bounds of  $G_{(n)}$  be arbitrarily chosen as  $g_1 = 0.5$ ,  $g_2 = 0.95$ , and  $g_3 = 0.99$ . Allow the number of simulations to vary from 1 to 150. The tradeoffs between the certainty that  $G_{(n)}$  exceeds its lower bound and the number of scenarios appear in figure 2.

FIGURE 2

#### TRADEOFFS BETWEEN RELIABILITY AND COMPUTATIONAL BURDEN



Each curve is the locus of points representing the most certainty that  $G(n)$  exceeds its designated lower bound for the given number of simulations. Consider the points along the curve associated with  $g_3 = 0.99$ . Any combination of certainty/number of runs to the right of the curve is feasible, but it would be dominated by at least one point on the curve itself.

For example, A is dominated by B because B achieves the same level of certainty that  $G(n)$  exceeds 0.99 with fewer simulations. Any point to the left of the curve is not feasible. Point C would dominate point B because it provides more reliability at a smaller computational burden, but point C is not feasible. The only way to increase the certainty that  $G(n)$  exceeds 0.99 is to increase the number of simulations evaluated for "worst" shortfalls, or to move from B towards point D.

Thus, the points on the curves and only those points will interest the analyst who wants to select a specific number of simulations. Any point represents the fewest scenarios that achieve the associated level of result reliability that  $G(n)$  exceeds its  $i$ th lower bound.

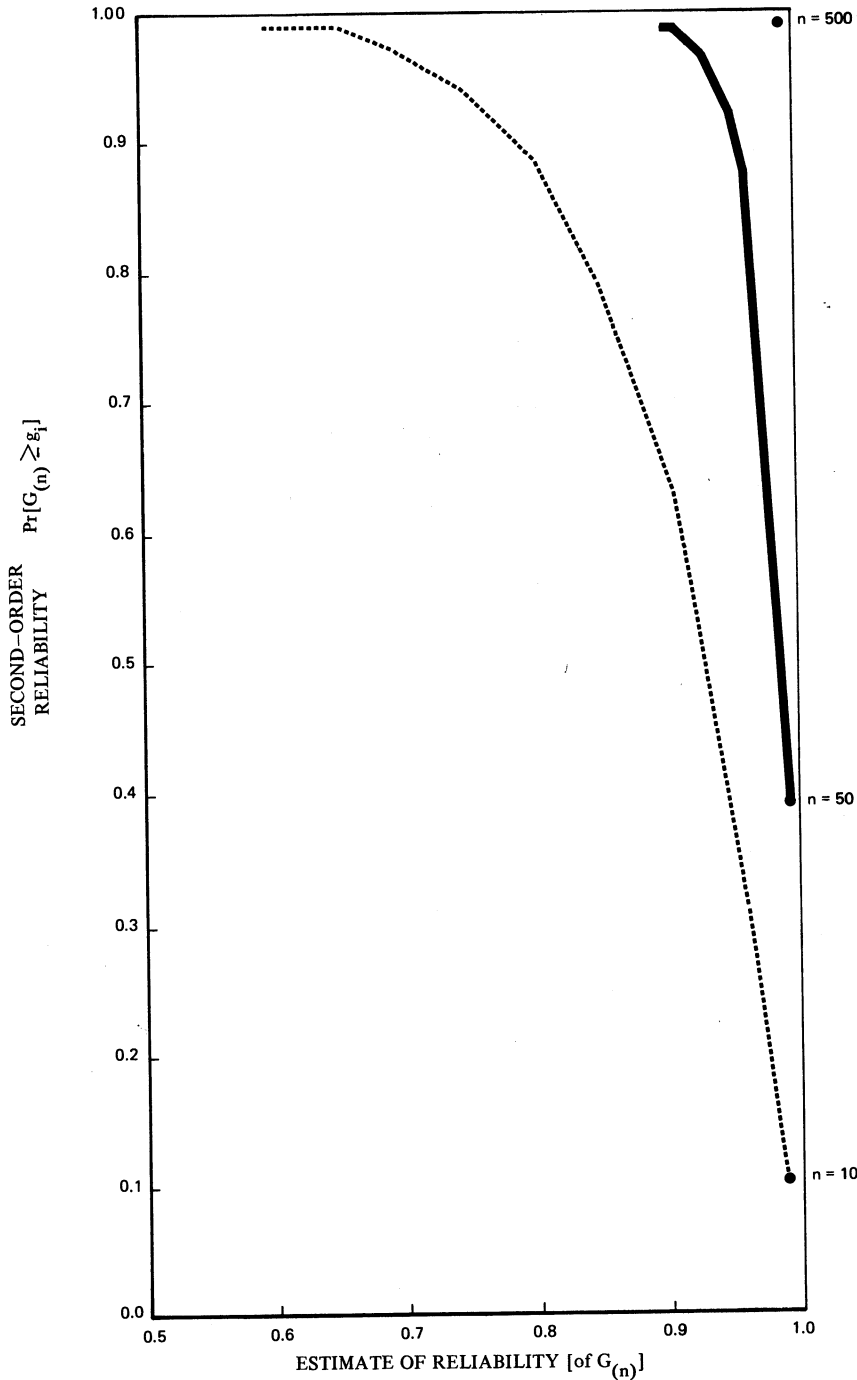
To illustrate these calculations, imagine that an analyst wishes to be 90 percent sure that the probability of a future shortfall exceeding the largest simulated shortfall is below 0.05. We can state this as  $\Pr[G(n) \geq .95] \geq .80$  and represent it by point E on figure 2. Even 160 simulations do not allow a statement that  $\Pr[G(n) \geq .99] \geq .80$ . Thus a 4-percent improvement from point E costs more than 400 percent in incremental computational burden. The absolute increases in computational burden may be tolerable at lower reliability levels. For example, one run is enough to assume that  $\Pr[G(n) \geq .50] \geq .50$ . The probability of achieving that result can be moved from 0.50 to 0.94 with only three additional simulations.

Another way to show the increasing marginal costs is to graph, for fixed numbers of design periods ( $n$ ), the tradeoff between first-order and second-order reliability, as in figure 3. Ten runs can assure a 0.6 reliability estimate with a probability of 0.99 or a 0.98 reliability with a probability of 0.10. An analyst would need 500 design period analyses to assure a probability above 0.99 that a reserve size C would perform at a food security level B at a reliability of 0.99.

The sample size of 34 used in chapter 8 is large enough to insure that the probability is less than 0.05 that the

FIGURE 3

TRADEOFFS BETWEEN FIRST- AND SECOND-ORDER RELIABILITY



, reserve capacities will fail their designated performance levels in one of ten design horizons. This performance level is the compromise used here between the unattainable quest of perfect reliability and the mundane consideration of computational burden. When results are described as "highly reliable" based on this sample, the meaning is implicitly a tradeoff curve (comparable to the curves of figure 3) for  $n$  equal to 34.

## USE IN POLICY ANALYSIS

The order statistics approach to evaluating the likelihood of grain shortfalls can be used as a technique by itself and as a tool for grain reserve policy analysis. As a method, it allows an analyst to speak about the probability of an extreme value of a random variable associated with a simulation technique.

The order statistics approach developed here can be applied to policy discussions of world or national grain reserve size. Imagine that some consensus would exist regarding an equation to simulate future production of grains. A policymaker could generate tens or hundreds of grain production data series that could, but need not, occur. Using the methods developed here, the policymaker could calculate and rank the largest aggregate sum of shortfalls from sequential lean years in each series and could identify  $Y(n)$ , the worst of all possible shortfalls.

$Y(n)$  represents an upper bound on the size of a grain reserve. If a grain reserve of size  $Y(n)$  would be established, it would be unlikely that a sequence of lean years would completely deplete the reserve. Indeed, if the aggregate total of shortfalls during these back-to-back lean years were to be withdrawn from the reserve, the sum of the needed grain would, by design, exactly equal the amount of the reserve. This upper limit is appropriate only if "supply stability" is the only system goal. If price stability, farmer or consumer gains, or economic efficiency are more important, the decisionmaker might prefer a smaller stock. The use of this method in evaluating world or national grain reserves may be fruitful for future research.

The order statistics approach does not consider the related problem of finding a reasonable operating policy that would trigger appropriate purchases to or releases of

grain from such a buffer stock. Techniques for such analyses are developed in the following chapter.

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## VII. — OPERATING RULES

When should grain be added to or released from a buffer stock? This chapter develops a procedure, based on implicit stochastic analysis, to determine and evaluate rules using price or volume triggers for operating a grain reserve. 16/

A comprehensive grain reserve policy is likely to specify operating rules and stock size decisions simultaneously. The procedures developed in this and preceding chapters involve three separate stages. First, determine optimal reserve size. Next, use the reserve sizing analysis to identify implicit decision rules. Finally, test these rules to evaluate performance.

The three-step procedure involves three analytical methods — optimization, regression, and simulation. The multiobjective linear program produces the raw data of operating rule analysis, the smallest reserve sizes  $C$  which achieve alternate levels of food security  $B$ . Regression analysis is used to isolate implicit decision rules from the collection of storage levels and related variables. Decision rules, once specified, are evaluated by observing their performance through simulated synthetic futures. The optimization and regression stages screen the range of possible operating rules for promising options. Simulation works by selecting from among the screened options a rule or rules that perform effectively.

Prior to the analysis stage, it will be useful to develop operational definitions of a decision rule and a normalized decision rule. A storage decision is defined as either an addition to or a release of grain from a buffer stock. Thus:

$$Q_t = S_t - S_{t-1} = I_t - U_t \quad t = 1, 2, \dots, n \quad (7-1)$$

where  $Q_t$  is the storage decision in year  $t$ ;  $S_t$  represents the level of storage in year  $t$ ;  $I_t$  is production of grain in year  $t$ ; and  $U_t$  is demand for grain in year  $t$ . A storage decision can be either positive or negative. If storage in year  $t$  exceeds the stock of the previous year, then  $Q_t$  exceeds 0. If storage in the previous year was larger, then  $Q_t$  is less than 0. Over the lifetime of a grain reserve,

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16/ Young, George K. "Techniques for Finding Reservoir Operating Rules." Ph.D. dissertation, Harv. Univ., Cambridge, Mass., 1966.

the series of storage decisions can be summarized in vector notation as:

$$Q_1 = Q_1, Q_2, \dots, Q_n \quad (7-2)$$

where  $Q_1$  is the vector of storage decisions over the first design lifetime.

Imagine many synthetic futures,  $\{m = 1, 2, \dots, f\}$ , each of which may be analyzed with the sizing technique. For each design horizon, one can identify a vector of optimal storage decisions,  $Q_m$ , composed of elements which select the smallest reserve size to meet a designated food security level for that synthetic future. To work with storage decisions which span both years of a synthetic future and alternate futures, it is useful to double subscript the storage decision variable as  $Q_{kt}$ , or in matrix form:

$$\bar{Q} = \begin{bmatrix} Q_{11}, Q_{12}, \dots, Q_{1t}, \dots, Q_{1n} \\ Q_{21}, Q_{22}, \dots, Q_{2t}, \dots, Q_{2n} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ Q_{k1}, Q_{k2}, \dots, Q_{kt}, \dots, Q_{kn} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ Q_{f1}, Q_{f2}, \dots, Q_{ft}, \dots, Q_{fn} \end{bmatrix} = Q_1, Q_2, \dots, Q_k, \dots, Q_f \quad (7-3)$$

where  $Q_{kt}$  = storage decision in year  $t$  in synthetic future  $k$

$Q_k$  = vector of storage decisions of design horizon  $k$ ; as in (7-2) where  $k$  was 1.

$n$  = total number of years of each synthetic future

$f$  = total number of synthetic futures analyzed

It is also useful to postulate a normalized storage decision, in which the actual decision is weighted by the associated optimal reserve size for the synthetic future. For the  $f$  synthetic futures, let  $C_k$  represent the optimal

reserve capacity in design horizon  $k$ , where  $k = 1, 2, \dots, f$ . Let  $S_{kt}$  be defined in an analogous way to  $Q_{kt}$ , to represent the level of storage associated with year  $t$  of design horizon  $k$ . The normalized storage decision  $Q_{tk}/C_k$ , measures the relative size of storage in year  $t$ . The actual decision is normalized by its associated reserve capacity. The variable  $Q_{tk}/C_k$  can also be viewed as the fractional or percentage increment in storage during year  $t$  of synthetic future  $k$ .

## SELECTION OF CANDIDATE RULES

A candidate storage rule is some procedure which is consistent with the observed optimal values of  $\bar{Q}$ , the vector of storage decisions. An analyst first seeks to specify variable(s) which could have influenced the storage decision. Next, the analyst posits some form of relationship between the vector of values of those variables and  $\bar{Q}$ . The strength and significance of the relationship is evaluated through regression.

A simple example of the many possible rule specifications is a linear relation between some variable and the storage decision in year  $t$ . An hypothesis that a storage decision should be a linear function of the volume in storage at the end of the previous year would be:

$$Q_{tk} = \psi_1 + \psi_2 + S_{t-1,k} \quad (7-4)$$

where  $\psi_1$  = intercept of the linear relation between  $Q_t$  and  $S_{t-1}$

$\psi_2$  = rate of change in the linear relation

$S_{t-1,k}$  = volume in storage at the end of the previous year

A linear normalized rule would be specified as:

$$Q_{tk}/C_k = \psi_1 + \psi_2(S_{tk}/C_k) \quad (7-5)$$

where  $\psi_1$  = intercept coefficient 17/

$\psi_2$  = rate of change coefficient 17/

Given either specification, regress the vector of the dependent variable ( $\bar{Q}$  or  $\bar{Q}/C$ ) against the vector of the independent variable. For equation (7-4), the storage decisions in any year  $t$  would be regressed against the volumes in storage at the end of the previous year. Such a regression would determine values for the intercept and rate of change coefficients. The coefficients and the degree of explained variance can be tested for statistical significance.

A candidate decision rule would be defined as a specification which explains a large fraction of the variance of storage decisions and has associated intercept and rate of change coefficients which are observed to be significantly different from zero. The normalized form of a rule is useful because it aims to explain decisions to add or release fractions of the reserve capacity and not simply volume amounts. Because different series of production may involve vastly different capacities, the normalized form may be more appropriate.

Two important kinds of rules are price and quantity triggers for a grain reserve. Such triggers state that when a decision variable measured in dollars or quantity reaches some level, the reserve responds with a prespecified action.

An example of a quantity rule would be this: When production in year  $t-1$  exceeds some target, purchase a specified volume of grain for the reserve; if production falls below a critical number, release grain from stocks. Independent variables which may be used in quantity triggers include past, present, or projected future production; past or present volumes in storage; past stock addition/release decisions; or past, present, and projected future grain volumes demanded or consumed.

Quantity triggers are most usefully phrased in relative terms. For example, a trigger based on previous year's harvest should not be keyed to the absolute volume but to the expected volume. Thus, a coefficient of glut or shortage may be defined as:

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17/ The intercept and rate of change coefficients must be of opposite signs to generate reserve additions and depletions.

$$\kappa_{kt} = I_{kt}/O_{kt}$$

$$t=1, 2, \dots, f$$

(7-6)

where  $\kappa_{kt}$  = coefficient of glut or shortage associated with year  $t$  in design horizon  $k$

$I_{kt}$  = production in year  $t$  of sequence  $k$

$O_{kt}$  = expected volume of production (also the expected volume of demand) in year  $t$  of synthetic series  $k$

Such normalized coefficients for other independent volume measuring variables could be developed. An example of an hypothetical quantity trigger is this: Let grain addition/release from a reserve be an exponential function of the glut/shortfall in production from the previous year.

A price trigger would state that when prices reach a ceiling level, specified stocks are released; when prices fall below a predefined floor, grain is purchased to add to the reserve. An example of a price band is this: Purchase wheat when market price falls below \$2 per bushel and sell wheat when costs are above \$5 per bushel. The multiobjective reserve models of chapter 5 use price in year  $t$  as a decision variable. Price levels in the same, previous, or projected future years may be used as independent variables in a regression against a vector of storage decisions.

The use of either price or quantity triggers implicitly assumes that the market responds to decisions about stocks. For example, when grain is released from stocks because of a ceiling price trigger, it is expected that added supplies will stabilize price levels. Such a view of market processes may be unrealistic, because psychological or speculative pressures that operate in real grain markets are not considered explicitly.

## EVALUATING RULE EFFECTIVENESS

Imagine that a candidate rule has been found. How can its effectiveness be evaluated? The reserve sizing models are collections of mathematical equations which can be used in multiobjective linear programming algorithms. The same equations can be used in a Fortran environment to simulate behavior of a reserve operating rule. Candidate rules could be simulated over many design lifetimes. The

reliability of a given rule in achieving some designated performance criterion could be evaluated by using the order statistics methodology of chapter 6.

## VIII. — AN EMPIRICAL EXAMPLE

The techniques developed in this study are now used to assess a world grain reserve problem.

### A GRAIN PRODUCTION GENERATING FUNCTION

A first step is to describe the volume of grain that might be produced in the world during 1975-2000. Let us generate a hypothetical time stream of production volumes, based upon some explicit rules. Assume that implicit in an historical series of production volumes is a production process — some pattern of growth and variability of production. 18/ Further, assume that the future will mimic the pattern. Table 10 contains USDA data on world grain production for 1960-74 (12). 19/

Table 10 — World grain production, 1960-74

Year	Production
	Million metric tons
1960	888.6
1961	862.4
1962	908.7
1963	910.7
1964	956.1
1965	952.2
1966	1,019.2
1967	1,060.8
1968	1,103.3
1969	1,118.0
1970	1,122.8
1971	1,211.6
1972	1,175.8
1973	1,282.8
1974	1,222.1

Source: U.S. Department of Agriculture, "Production Data Base," unpublished computer printout of world production statistics 1960-74, Washington, D.C., 1975.

18/ Such rules may or may not be reasonable, but at least they are explicit and results would be replicable. See chapter 3 for a discussion of alternative approaches for developing representations of time streams of future production.

19/ Total grains include wheat, corn, rice, sorghum, oats, rye, barley, and miscellaneous food and feed grains. The U.S. Department of Agriculture series was chosen for use over a data series from the Food and Agriculture Organization of the United Nations because the USDA data were more accessible.

How can a pattern of past (and presumed future) production behavior be isolated from this data series? The expected volume, variance, and lagged covariance behavior of the series will be characterized by analytical steps of assumption, curve fitting, and statistical analysis. This information will be incorporated in a function designed to mimic that behavior.

### Expected Volume and Variance

Grain production increased between 1960 and 1974 (table 10 and figure 4). Assume that the expected crop size can be characterized as a function of time, or as some "trend" volume:

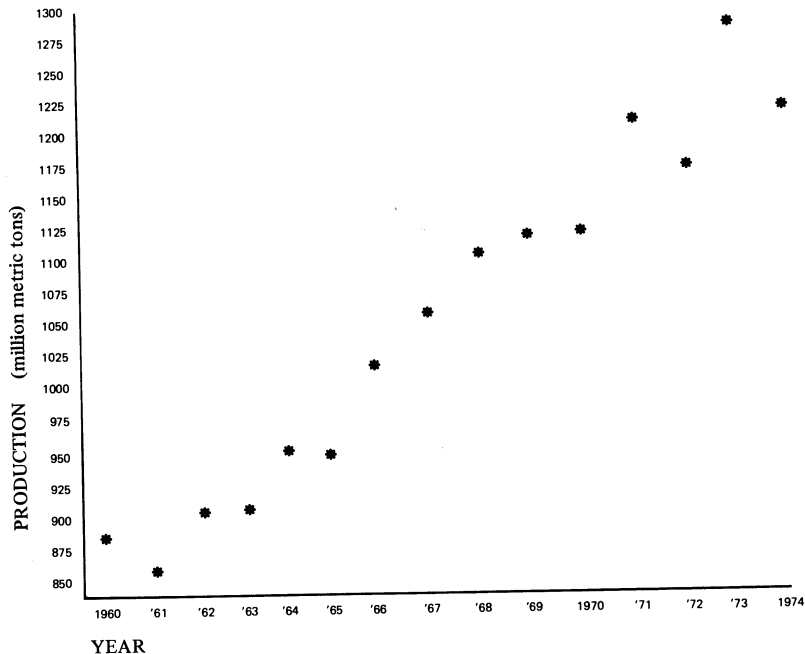
$$E[I_t] = f(t) \quad (8-1)$$

where  $E[I_t]$  = expected production in year  $t$

$f(t)$  = a function of time

FIGURE 4

#### WORLD GRAIN PRODUCTION, 1960-74





It is possible to find a mathematical equation which approximates the historical record closely by fitting various curves (corresponding to alternative functions) to the historical data series. For example, Malthus described 18th century crop production as a linear trend (7). Other investigators have found an exponential curve satisfactory for recent crop production series (11, p. 14; 6, p. 63).

Such curve fitting to a production series implies that the analyst accepts these working hypotheses:

- . The data series characterizes how crop production behaves over time,
- . Physical factors — climate, land fertility, acreage tilled, water and fertilizer supplies, and labor — need not be modeled explicitly but are incorporated in the data series,
- . Government policies, prices, technological change, and political/economic system are not described but that they influence the historical record is recognized.

These assumptions are not intended as statements of truth; they are working suppositions made to conduct an analysis. Thus, they should be stated explicitly. Other investigators may wish to work with other assumptions.

Linear and exponential trends were determined by fitting annual world total production data to linear and exponential functions of time. While both functions were found satisfactory statistically, the linear trend is used because it is simpler to work with.

World production was regressed against time to determine least squares estimators for the rate of change and intercept for the assumed linear trend (9):

$$I_t = -935.888 + 28.685 t \quad (8-2)$$

where both numbers are expressed in million metric tons and  $t$  is an index of the year, represented by final two digits of a calendar year; for example,  $t$  equals 63 for

1963. 20/ To test the significance of these results, divide each least squares estimator by its associated standard error to yield a t-statistic. These were compared to reference values (10, pp. 617-627), to test the null hypothesis that the estimators are not significantly different from zero (table 11). As a test of the hypothesis that the regression results are explained by chance, the F statistic is evaluated. These results support the working hypothesis of a linear trend.

Table 11 — Regression results

Variable name	Least squares estimator	Standard error	Associated statistic type or value	Level of significance
Intercept	935.888	120.42	-7.772	0.0005
Slope	29.685	1.79	16.551	0.0005
Fraction of variance explained	0.9547	30.012	F:237.94	0.001

The specification of a trend function also determines one measure of the variability of production about the expected volume. The differences between actual and estimated (trend) production for the 15-year series are a sample from which a variance of production fluctuations can be calculated — the standard error of the estimate (30.012 million metric tons (MMT)). This measure of variability implicitly accepts the notion that the magnitude of fluctuations of production about trend remains constant over time. 21/

20/ This form of a regression equation may be puzzling. The intercept is a negative number, which seems to imply that world grain production back in the early 20th century was negative. The regression used the data series of world production in table 10. Its results should not be applied to years prior to 1960.

21/ This is the assumption of homoscedasticity, or constant variance. If an analyst wishes to accept an alternative hypothesis about the behavior of fluctuations over time (that they increase or decrease), the standard error of the estimate may not be the appropriate statistic to describe the fluctuations of the stochastic process. An empirical test of homoscedasticity, a constant coefficient of variation, or some other hypothesis would not mean much, given the short length of the data series.

This study assumes homoscedasticity, or constant variance, rather than an increasing magnitude of fluctuations over time. This assumption leads to smaller fluctuations during 1975-2000; hence, to a smaller grain reserve requirement to stabilize supplies during those fluctuations. This procedure follows the pattern described in chapter 1, of consciously accepting assumptions (when required) which lead to a lower level of reserves than the realistic alternatives. 22/

### Lagged Covariance Structure

Another statistical issue is whether surplus or shortage of production in any year affects production in another future year — the lagged covariance behavior of the process. The hypothesis of this study was that we might observe a behavior consistent with a cobweb effect — negative serial correlation of lag one. This would implicitly reflect the effect of price expectations on production. The current price of a crop is determined by the volume of this year's supplies; the size of the current crop is a function of last year's price (4). 23/

Production fluctuations in each of the years were regressed against the deviations of the previous year, of 2 years before, 3 years before, and so on, back for seven periods. 24/

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22/ The purpose is to find a lower bound on the size of global grain reserves. The notion of decreasing variability of grain production in the future is not considered a reasonable alternative to stable or increasing magnitudes of fluctuations.

23/ Alternative assumptions were discussed in chapter 4.

24/ The computation package used (8) bases calculations on the theory of (1). Given the brevity of the data series (15 years), even a single autocorrelation coefficient is based on a minute sample of 14 observations. The calculations of the autocorrelation and partial autocorrelation coefficients of lags greater than seven would have been based on samples of seven or less observations. The reason for going as far as (or stopping at) seven was to see whether 5- or 6-year cycles of grain production would be observed (2, p. 9; 3, p. 184).

$$D_t = f(D_{t-1} \text{ or } D_{t-2} \text{ or } D_{t-3} \text{ or } D_{t-4} \text{ or } D_{t-5} \text{ or } D_{t-6} \text{ or } D_{t-7}) \quad (8-3)$$

where  $D_t$  is the deviation of actual production from the expected volume or  $D_t = I_t - E[I_t]$ .

The seven autocorrelation coefficients, one for each lag, represent the autocorrelation function. The effects of one lag effect on the measurement of another (for example, a lag one effect on measurement of a lag four effect) were removed to yield seven partial autocorrelation coefficients. Standard errors for the auto- and partial auto-correlation functions were also calculated (table 12).

25/

Table 12 – Autocorrelation statistics and significance

Lag number	Autocorrelation function		Partial autocorrelation function	
	Sample estimator	Standard error	Sample estimator	Estimate of standard error
1	-0.43	0.26	-0.43	0.27
2	0.38	0.30	0.24	0.28
3	-0.34	0.33	-0.14	0.29
4	0.04	0.35	-0.25	0.30
5	-0.08	0.35	-0.02	0.32
6	-0.18	0.36	-0.28	0.33
7	0.07	0.36	-0.17	0.35

Visual inspection of the pattern of auto- and partial auto-correlation coefficients suggest the possibility of a negative lag-one autoregressive process. The evidence includes:

- Alternating positive and negative autocorrelation coefficients in the first five periods which diminish in absolute magnitude,
- Size of the lag-one auto- and partial autocorrelation statistics; 26/

25/ Estimates of the standard error of the partial auto-correlation function were calculated as  $(1.0/\text{square root of } n)$ , where  $n$  is the sample size.

26/ The lag-one autocorrelation coefficient (-0.43) is associated with a t-statistic of 1.654. This is significant beyond the 0.1 level and, by linear interpolation, beyond the 0.07 level.

- No other partial autocorrelation coefficient as large as the standard error estimate.

This evidence, combined with the finding that the entire autocorrelation function differs from the behavior of a random process, provides empirical support for a cobweb hypothesis. <sup>27/</sup> The behavior that would be observed if production was responding to price expectations would be a lag-one negative autoregressive process.

### Grain Production Generation

Chapter 4 contained equations which can generate series of random numbers which conserve prespecified statistical properties, such as expected value, variance, and lagged covariance behavior. One function which increased over time along a linear trend path, permitted negative lag-one autoregressive effects, and included random normal fluctuations of stable variance was equation (4-16):

$$I_{t+1} = a + b(t+1) + r(I_t - a - bt) + \ell_{t+1} s(1 - r^2)^{0.5} \quad (8-4)$$

where  $I_{t+1}$  = production in year  $t+1$

$a$  = estimate of intercept in linear trend

$b$  = estimate of rate of change in linear trend

$r$  = lag-one autocorrelation coefficient estimator

$s$  = estimate of standard deviation

$\ell_{t+1}$  = pseudo-random deviate, generated for year  $t+1$

Estimates of each parameter have been made in preceding pages of this chapter. When the values (table 13) are

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<sup>27/</sup> The chi-square statistic for the autocorrelation function is 2.722. This statistic is compared with a reference value to see whether the calculated coefficients could easily have been generated by production volumes that are drawn from a distribution of independent random normal events. The chi-square statistic is significant beyond the 0.1 level. This level of significance is open to varying interpretations, but it is high enough, given the length of the data series, to be consistent with a cobweb process.

substituted in equation (8-4), it becomes a generating function of world total crop production for 1975-2000. Any number of synthetic series can be generated, each with an expected volume, standard deviation, and lag covariance behavior denoted in table 13.

Table 13 - Estimators

Variable	Symbol	Value
		<u>Million metric tons</u>
Intercept of linear trend	a	- 935.888
Rate of change of linear trend	b	29.685
Standard deviation	s	30.012
		<u>Number</u>
Autocorrelation coefficient (1st order - lag one)	r	-0.43

The final generating function appears as:

$$I_{t+1} = -935.888 + 29.685 (t + 1) + (-0.43)[I_t - (-935.888) - 29.685 t] + \epsilon_{t+1} (30.012)[1.0 - (-0.43)^2]^{0.5} \quad (8-5)$$

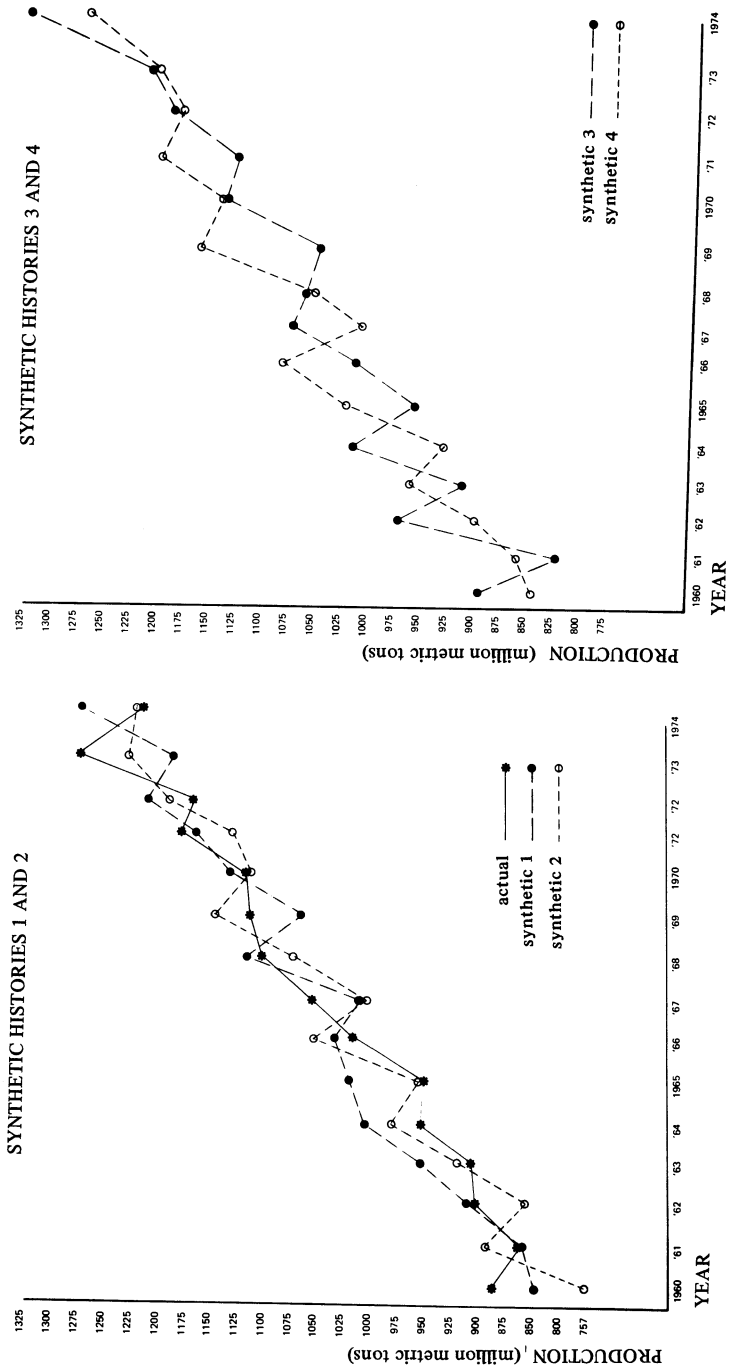
This function can be used to generate synthetic futures for 1975-2000 or to generate synthetic histories for 1960-74. Figure 5 shows four such synthetic histories of world total grains production for 1960-74 compared with the actual volumes produced. This function (8-5) can generate any number of synthetic histories which, while different from each other and the observed time series, maintain an identical expected volume, variance, and lagged covariance behavior with the historical production series.

#### THE RESERVE SIZING PROCEDURE

Figure 6 is a flow chart of methods used to size a grain reserve. First, the historical series of fluctuations of grain production are characterized. Next, the analyst makes explicit assumptions concerning institutional issues,

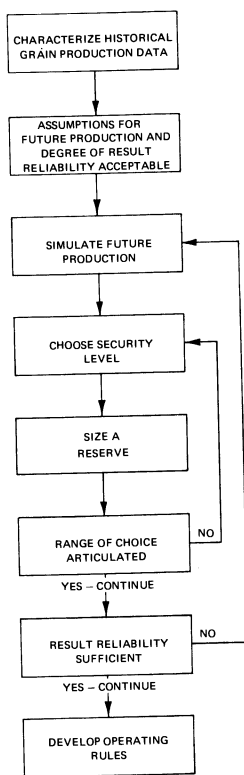
FIGURE 5

SYNTHETIC HISTORIES OF WORLD TOTAL GRAINS PRODUCTION, 1960-74



the acceptable level of reliability of results, and the relationship of future to past production.

FIGURE 6  
GRAIN RESERVE ANALYSIS



Synthetic futures can now be generated and, for a designated food security level, the size of a grain reserve found. The procedure then asks whether the full tradeoff curve between food security and reserve size has been articulated. If not, a new security level is chosen and the reserve sizing step repeats.

If all feasible alternatives have been generated, the method inquires whether a sufficient number of design horizons have been studied to justify the preselected reliability level. If not, another synthetic future can be generated and the analysis continues. If the reliability level is sufficient, the program terminates. Several of these steps require further discussion: the assumptions, reliability estimates, food security levels, and size of the reserve.



The analyst must make explicit assumptions before proceeding. To use the synthetic futures technique, the analyst must assume that future production extends the historical pattern, so that volume, variance, and lagged-covariance behavior remain the same.

Seven institutional issues must be specified before analysis begins: the type, scale, and scope of the stock, and the degree of grain substitution, trade, access, and transportation. The focus here is a world, total grains buffer stock. The aim is to calculate a lower bound on reserve size. Assumptions include these — perfectly free trade among nations, perfect substitution in use of grains, and no barriers to transportation or human access to grain. We can then calculate the smallest reserve that could be calculated based on any assumptions possible for the issues of grain trade, substitution, transportation, and access.

One additional assumption is the degree of result reliability deemed acceptable. A result will be deemed reliable here when the probability is less than 0.05 that the reserve capacity will fail to achieve a designated performance level in one out of 10 design horizons. This corresponds to a requirement that sizing be based on results from 34 synthetic futures. To be more explicit, recall that reliability is also a random variable whose range can be limited through second-order probability statements. The expected value of the reliability of a reserve to perform at designated food security levels is 0.971, based upon a sample of 34 future investment periods.

The chance that the real reliability exceeds three arbitrary estimates of reliability (0.9, 0.95, and 0.99) is:

$$\Pr [G_{(n)} \geq 0.90] = 0.972 \quad (8-6)$$

$$\Pr [G_{(n)} \geq 0.95] = 0.825 \quad (8-7)$$

$$\Pr [G_{(n)} \geq 0.99] = 0.289 \quad (8-8)$$

Thus, when the term "reliable" is used, it means that:

- The chance that a reserve designed on the basis of 34 design horizons will fail is on the order of 2.9 percent (expected value of reliability),

- . The probability that such a reserve will function with better than 90 percent reliability is 0.972,
- . The probability that the reserve will be more than 95 percent reliable is 0.825.

Food security, as defined in chapter 1, is the extent to which a stock stabilizes food supplies through years of good or lean production. The test of security is that the reserve be larger than the cumulative deficits from the worst series of back-to-back lean years that could occur based on existing information (such as the extreme event of shortfall).

When a reserve capacity exceeds the extreme shortfall event, the stock may be said to provide 100 percent food security; consumption in any year can equal expected volume of demand of that year. If a stock is completely depleted in 1 year of the reserve investment lifetime, and in that year only 99 percent of expected demands can be met, food security performance is defined as 99 percent.

This study describes the full range of physical feasibility, from complete food security to a level no greater than that achieved by the unbuffered fluctuations of production. This range is operationally defined as food security levels of 94 to 100 percent security, taken in increments of 0.5 percent.

The author generated 34 synthetic futures of world grain production for 1975-2000 and used each as an exogenous input for the production vector in the two-objective reserve sizing formulation of table 5. B (lower bound on food security) was varied from 0.9400 to 1.0050 in increments of 0.0050. The MPSX linear programming algorithm of International Business Machines, Inc. (IBM) was used to solve the linear programs. The average cost per solution was 0.17 second or 21.6 cents on an IBM 360/370 computer system.

## BUFFER STOCKS ANALYSIS RESULTS

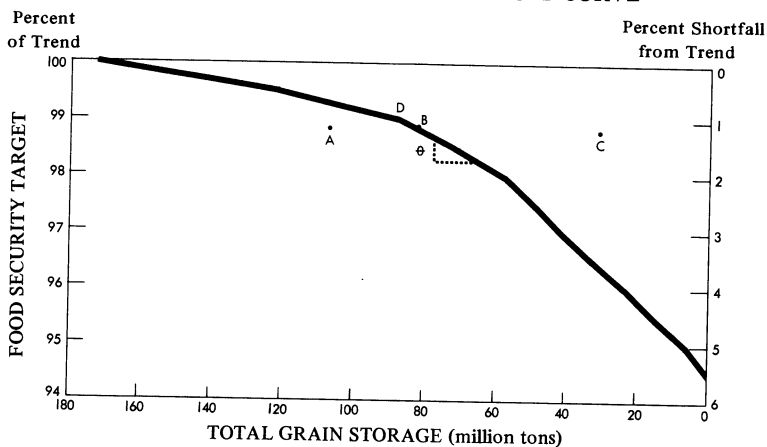
A series of tradeoff curves — how much of one objective need be sacrificed to achieve a higher level of a conflicting objective — report the results. Such curves show the full range of physical feasibility and the implications of adopting any particular priority ranking among objectives.

The curve in figure 7 shows the tradeoff between food security and the reserve size. The tradeoff between capa-

city and security is that greater security can only be gained at the expense of a larger buffer stock. Note that capacity decreases from the origin.

FIGURE 7

### A RELIABLE\* CAPACITY-FOOD SECURITY TRADEOFF CURVE



This curve developed for the period 1975-2000.

\*See text for a statement on the degree of reliability.

This tradeoff curve shows the boundary of the feasible region. All points inside (to the left of) the curve are feasible, but they are dominated by at least one point on the boundary (the curve). All points outside (to the right of) the curve are not feasible. All points on the curve are feasible and are not dominated by any other feasible points; it is impossible to improve unambiguously along one objective without a sacrifice in the value of the other. For example, point A is feasible, but it is dominated by point B, because point B achieves an identical level of food security at a lower storage volume. Point C would dominate point B, because C achieves an identical level of food security at a lower capacity, but point C is not physically feasible. Points B and D are part of the noninferior set, the tradeoff curve; food security improves only at the cost of increased buffer stock capacity. Thus, the points on the transformation curve, and only those points, are of interest as policy alternatives.

The choice of one of the boundary points as an alternative for implementation implies the imposition of a set of preferences for one objective relative to the other. The implicit tradeoff can be made explicit by drawing a tangent line to the curve at the point selected. Such a tangent line has been drawn at point B on figure 7. In general,

when the slope of the tangent line to the transformation curve at a point is  $-\theta$ , the implicit tradeoff between objectives is  $\theta$ . That is, it is implied that 1 percent of food security can be (should be) sacrificed for a decrease in required capacity of  $\theta$  MMT. For point B on figure 7, 1 incremental percent of food security is worth 32 MMT of additional buffer stock capacity.

At any corner point, such as D, we cannot state the tradeoff, because the tangent at a point such as D is undefined. A range of tradeoffs can be defined for such extreme points, which corresponds to the slopes of the line segments to either side. For point D, this range is between 32 and 66 MMT per marginal percent of food security.

### Reserve Size Versus Food Security

The curve in figure 7 provides much information useful for policymakers. For a designated reliability, it shows how reserve capacity must be increased to gain any desired level of food security. The capacity-security tradeoff varies along the curve. Moving from 95- to 96-percent levels of security requires 17 MMT of additional storage. Covering the final incremental percent, from 99 to 100, requires 84 MMT. As might be expected, the final percentage increments cost more than the first ones.

The food security levels achieved by alternate grain stock sizes are marked "reliable" in figure 7. The implication is that any reserve size below these values will not meet the designated food security target with such high reliability. It cannot be determined from these data how much larger a reserve would need to be if one or several of the lower-bound assumptions were relaxed.

For example, grant that a public goal be to provide at least 98 percent of the trend production/trend demand for all years of 1975-2000. A reserve of at least 58 MMT would be required to achieve the 98-percent target level in figure 7. The tonnage needed is less than 5 percent of total world grain production (12). A buffer stock less than 58 MMT would not be as likely to buffer supplies within 2 percent of trend. Such an amount would meet the 98-percent target reliably under the restrictive assumptions of this exercise. It would likely fail if the lower-bound assumptions were relaxed: a single, global, total grain buffer stock; free trade and perfect substitution among grains; and no barriers to transportation or distribution.

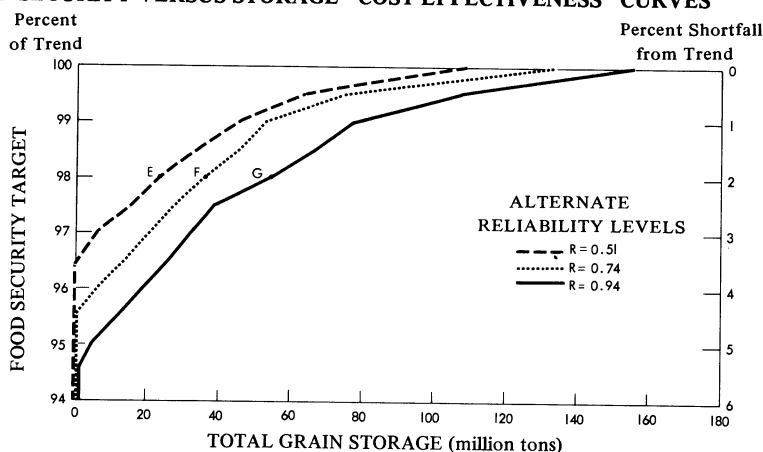
Results of relaxing the lower bound assumptions are an important item for future research.

### Size Versus Reliability Versus Food Security

The tradeoff curve between food security and buffer stock size relates to the level of desired reliability. Three tradeoff curves in figure 8 correspond to three reliability levels. These curves show the effectiveness (as evaluated in terms of reliability and security) of additional grain stocks.

FIGURE 8

#### FOOD SECURITY VERSUS STORAGE "COST EFFECTIVENESS" CURVES



\*Each alternate reliability level refers to an estimate of the probability that a stated reserve size will meet the designated food security target during 1975-2000.

R is defined as the expected value of the reliability, the probability that each capacity on the given curve will perform at its associated level of food security. For example, if R equals 0.51, then, in one of two synthetic futures, a reserve sized according to the points on the curve will fail to meet the designated level of food security.

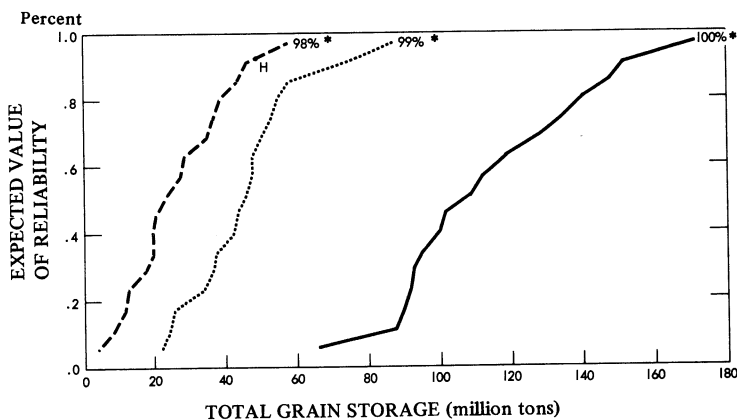
As the estimate of reliability increases, the tradeoff curve shifts downwards and to the right. This movement reflects the sacrifice in the level of food security required to boost reliability. For example, a 98-percent target of food security can be met with only 24 MMT of stocks with an expected reliability of 0.51 (point E on figure 8). To increase the reliability of performance to 0.74 requires

37 MMT of stocks (point F); to 0.94, a reserve of 55 MMT (point G). A 129-percent increase in stock capacity is required to boost reliability 85 percent (from 0.51 to 0.94) while maintaining food security performance at 98 percent.

The results embodied by the curves of figure 8 can be presented in a different manner that emphasizes the food security or reserve size sacrifices required for greater reliability. Figure 9 show tradeoff curves between reliability and reserve size at three fixed food security levels: 98, 99, and 100 percent. The percentage values represent the minimum fraction of trend production/demands that can be made available in all years from 1975 to 2000 through the supply stabilization effects of a buffer stock. The tradeoff curves of figure 9 are different from those of figures 7 and 8 because the expected value of reliability is not an objective (in the mathematical programming sense). While reliability may be a goal, it is calculated as an account and not optimized formally. Hence, the curves are not sets of noninferior points.

FIGURE 9

#### RELIABILITY VERSUS STORAGE "COST EFFECTIVENESS" CURVES



\*Each value of percent refers to a level of food security coverage (expressed in terms of trend production), during 1975-2000.

As the level of food security increases, there is a rightward shift in the curve which measures the tradeoff between reliability and storage capacity. Meeting a larger fraction of trend demand at a constant reliability requires a larger grain reserve. For example, a 41 MMT stock is sufficient to provide more than 98 percent of expected demand with an expected reliability of 0.9 (point H). The same size of reserve is completely unreliable in performing at a 100-percent food security level. A gap of 15-20 MMT separates equally reliable performances for the 98- and 99-percent food security targets. Moving the final incremental percent, from 99 to 100, costs more; the gap is between 45 and 60 MMT.

The costs of greater reserve reliability appear in figure 9. On the 99-percent curve, increasing the expected value of reliability from 0.35 to 0.87 requires a 20-MMT increase in stocks, from 40 to 60. Achieving very high reliability costs more on a marginal basis. To move from 0.87 to 0.97 on the 99-percent curve requires 29-MMT more grain.

#### MULTIDIMENSIONAL DISPLAY OF TRADEOFFS

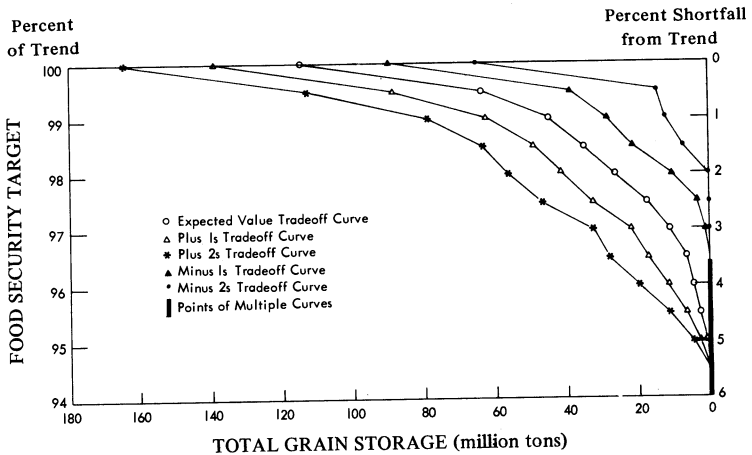
We must now decide how to display results because there are more than two dimensions. The example has two objectives but three dimensions — reserve size, food security, and reliability. The tradeoff curves of figures 7 through 9 are two-dimensional cross-sections of a three-dimensional surface.

A tradeoff curve between food security and capacity can be graphed in two dimensions if the reliability level is fixed. Thus, such tradeoff curves exist only in a probabilistic sense. This resembles the concept of an electron, which cannot be found, but the likelihood of its existence in some region around the atom can be computed.

One way to visualize the indeterminacy of the location of a tradeoff curve is to graph its probability distribution. Figure 10 is a two-dimensional projection of the three-dimensional distribution of the tradeoff curve between reserve capacity and food security. The middle curve, designated with small circles, is the expected value-tradeoff curve; this is the most likely site for the curve based on the sample of 34 reserve design horizons. To the right and left are curves representing curves one and two standard deviations from the expected curve.

FIGURE 10

## A PROBABILISTIC CAPACITY-FOOD SECURITY TRADEOFF CURVE\*



\*For the design period 1975-2000.

Such a set of curves is difficult to interpret. If the analyst has determined or is willing to assume that the tradeoff curves follow some known distribution, it becomes possible to make statements regarding their location. For example, if tradeoff curves follow a normal distribution, a tradeoff curve would appear beyond the boundaries of the +2s and -2s tradeoff curves in less than 5 percent of the synthetic futures (0.045) (fig. 10).

Figure 10 shows how murky conclusions would become without the order statistics approach. Given the nonparametric approach used here, the distribution of tradeoff curves does not matter. We can graph a highly reliable boundary to the region wherein the tradeoff curve will lie (as in figure 7).

## COMPARISON OF RESULTS WITH JOSEPH'S RESULTS

Complete and highly reliable stabilization of grain supplies (100-percent food security) requires a buffer stock of at least 172 MMT (figure 7), a quantity that is 14 percent of total grain production in 1974 (12). It is also 5.7 times the standard deviation of production fluctuations about trend. With no buffer stock, supplies could fall in some year between 1975 and 2000 to 5.5 percent below the



expected volume of demands. These results are based on assumptions which lead to the calculation of a lower bound on world buffer stocks.

Joseph saved 20 percent of the total annual grain production in Egypt for a reserve in each of 7 productive years. Production in each of those years, according to accounts, exceeded the expected volume. The aggregate reserve was probably far greater than 140 percent of the annual production of total grains in Egypt. Thus, Joseph's reserve was proportionally at least one order of magnitude larger than the maximum reserve size suggested here to stabilize supplies of grain.

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